Platform governance*

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Abstract

Platforms that intermediate trades such as Amazon, Airbnb, and eBay play a regulatory role in deciding how to govern the "marketplaces" they create. We propose a framework to analyze a platform's non-price governance designs and its incentive to act in a welfare-enhancing manner. We show that the platform's governance designs can be distorted towards inducing insufficient or excessive seller competition, depending on the nature of the fee instrument employed by the platform. These results are illustrated with micro-founded applications to a platform's control over seller entry, quality standards, and search design choices.

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1 Introduction

A growing number of platform intermediaries run "marketplaces" through which buyers and sellers trade. Well-known examples include third-party marketplaces Amazon and eBay, accommodation sharing site Airbnb, video game console such as Sony PS4, as well as brick-and-mortar shopping malls. Much like a government regulator that runs an economy, a platform regulates the behavior of platform users through its governance designs. Interpreting a platform this way highlights its governance role beyond just setting prices, as pointed out by the EU Competition Policy Report (Crémer, de Montjoye, and Schweitzer, 2019):

> "*Platforms play a form of regulatory role as they determine the rules according to which their users, including consumers and business users, interact... Moreover, we would expect that in many cases, platforms have incentives to write good rules to make their platform more valuable to users. However, this might not always be the case...*"

Given the role of a platform to govern participants in its marketplace, a natural question is whether a profit-maximizing platform will act in a welfare-maximizing way. In this paper, we provide a framework to examine the incentives and trade-offs that a platform faces in its governance designs, and study how its designs can be distorted away from what is optimal for the entire marketplace or from a total welfare perspective.

In practice, the scope of platform governance is wide. We focus on governance designs that have the following primary features: (i) they (indirectly) influence the gross transactional value generated on the platform for buyers, and (ii) they influence the extent of on-platform seller competition. These include decisions regarding entry, quality control, and on-platform search friction. For example, shopping malls regulate the number of competing tenants by carefully selecting which retailers to accept for each category as well as specifying their minimum opening hours. Airbnb implements quality control by continuously tweaking its peer-review system to induce honest user feedback and removing listings subject to excessive user complaints, while similar practices are used by Amazon and eBay. In addition, these platforms often actively redesign their user interface and search algorithm to facilitate the user experience, which ultimately alters the on-platform search friction faced by consumers.

Using our framework, we compare the optimal governance design by a profit-maximizing platform against the governance design maximizing welfare in order to examine the source of welfare distortion in platform governance. We identify a class of models where the sign of the welfare distortion can be related precisely to the fee instrument employed by the platform. In our framework, there is a platform that facilitates transactions between buyers and price-setting sellers, and it chooses its governance design and the fee(s) charged to sellers. Each buyer wants to buy one unit of a single product and purchases it from the seller of its choice. We initially focus on the case in which the platform fee is purely transaction-based, which can be either a proportional fee or a per-transaction fee.

With proportional fees, which are used by many different online platforms, we show that the platform's profit can be written as a weighted sum of seller profit and transaction volume, and so its governance design aims to balance the interest of the two parties. When the sellers have low marginal costs, the platform's profit — which is proportional to seller revenue — approximates seller profit so that the platform benefits from a governance design that relaxes seller competition and sustains a high markup for sellers. Therefore, the profit-maximizing governance design is distorted towards relaxing on-platform seller competition as compared to the welfare benchmark. However, as the sellers' marginal cost increases, the platform's profit begins to diverge from seller profit given that it does not internalize sellers' marginal cost. Once the marginal cost is sufficiently high, the platform's incentive is reversed and it now prefers to set a governance design that maximizes transaction volume, so that its design becomes distorted towards intensifying seller competition instead. We further show this distortion can lead to insufficient gross transactional value generated for buyers, depending on the correlation between the extent of on-platform seller competition and the transactional value that is induced by a change in the platform's governance design.

With per-transaction fees, the platform's profit increases with total transaction volume so that it sets its governance design to maximize this volume. This over-emphasis on transaction volume means that the platform can potentially fail to balance the dual roles of governance (i.e. influencing transactional surplus generated and affecting the extent of on-platform competition) in a welfare-maximizing manner. Indeed, we find that the profit-maximizing design is, in general, distorted towards intensifying on-platform seller competition as compared to the welfare benchmark. This distortion arises provided the correlation between the extent of on-platform seller competition and the transactional value for buyers is sometimes negative.

We then extend our analysis by allowing the platform to charge participation fees on sellers, e.g. listing fees that many online marketplaces charge on sellers. With pure participation fees, the platform profit becomes proportional to the joint industry (the platform and sellers) profit, and its governance design is distorted towards relaxing on-platform seller competition to maximize this joint profit. A similar intuition applies when the platform charges seller two-part tariffs (i.e. when both transaction-based fees and participation fees are feasible).

Our results thus motivate the following taxonomy for platform fee instruments (or revenue-generating models in general). On one hand, there are volume-aligned fee instruments (e.g. per-transaction fee, and proportional fee when seller marginal cost is low), in which the platform prefers governance designs that intensify seller competition and increase transaction volume. On the other hand, there are *seller-aligned* fee instruments (e.g. proportional fees when seller marginal cost is high, seller participation fees, and two-part tariffs), in which the platform prefers governance designs that relax seller competition and increase seller surplus instead. This taxonomy provides a simple way to relate platform fee instruments with the directions of welfare distortion in platform governance that our framework identifies.

The above results are established in a general setting without imposing any specific model of seller competition or platform governance. To illustrate the implications from the analysis, we then apply our framework to three applications of platform governance:

- Platform regulation of seller entry in the discrete choice model of Perloff and Salop (1985): Restricting entry relaxes seller competition, implying that a platform with seller-aligned fee instruments may over-restrict seller entry (relative to the welfare benchmark) to sustain seller markups.

- Imposition of a minimum quality standard in the consumer search environment of Eliaz and Spielger (2011): Raising the quality standard intensifies seller competition because a better seller quality pool means that consumers spend less time searching for good-quality products, which effectively means a lower search cost and hence a more price-elastic demand for each seller. The result suggests that a platform with seller-aligned fee instruments will impose a quality standard that is too low to sustain seller markup, while a platform with volume-aligned fee instruments will impose the highest quality standard (consistent with the welfare benchmark).

- On-platform search friction: A design choice that reduces search cost will intensify seller competition in the random search model of Wolinsky (1986), but it may relax seller competition in the price-directed search model of Choi et al. (2018). This means that whether a platform has excessive or insufficient incentive to reduce search costs depends not only on the fee instruments employed, but also on the search environment considered.

The rest of the paper proceeds as follows. Section 1.1 surveys the relevant literature. Section 2 lays out a general framework that nests various models of seller competition and platform governance models. Section 3 and Section 4 analyze the general framework under a variety of platform fee instruments. Section 5 applies the insights obtained to discuss specific models of platform governance. Section 6 discusses model extensions. Finally, Section 7 concludes. All omitted proofs and derivations are relegated to the Appendix.

1.1 Relation to the literature

Most of the existing literature on multi-sided platforms has been focused on pricing aspects (Caillaud and Jullien, 2003; Rochet and Tirole, 2003, 2006; Armstrong, 2006; Hagiu, 2006; Armstrong and Wright, 2007; Damiano and Li, 2007; Weyl, 2010; Jullien and Pavan, 2019). Our work contributes to the recent efforts that expand the formal study of multi-sided platforms beyond pricing into the domain of platform governance. Among the platform governance design decisions investigated in the strategy and economics literature are: platform ownership (Nocke et al, 2007), platform openness and innovation (Boudreau, 2010; Parker and Van Alstyne, 2018), intellectual property sharing (Niculescu, 2018), introduction of platform first-party content (Hagiu and Spulber, 2013), and delegation of control rights (Hagiu and Wright, 2015a; 2015b; 2018). Most of these works focus on how the platform governance designs help to generate additional surplus on platforms by encouraging innovations by third-party developers or coordinating end-user behavior. Except Nocke et al. (2007), these works do not investigate the role of governance in influencing on-platform price competition between sellers. On the other hand, they explore interesting innovation and coordination decisions faced by platforms which our framework of governance does not capture.

The focus on on-platform seller competition is also at the heart of Nocke et al. (2007), Belleflamme and Peitz (2019), and Karle et al. (2019). In these papers, membership pricing by platforms affects the number of participating sellers hence the extent of seller competition. In equilibrium, different levels of seller markup and market outcomes are induced by platforms' pricing, depending on various exogenous factors such as platform ownership, the strength of cross-network effect, or the extent of product differentiation among sellers. In contrast, the current paper explores how the fee instruments employed by the platform shape its incentive in governance designs.

Our emphasis on the role of different platform fee instruments (in particular, transaction-based fees) relates to the works by Shy and Wang (2011), Johnson (2017), and Wang and Wright (2017), among others. These works compare constant per-transaction fees against proportional fees, and they show the superiority of the latter in mitigating the double marginalization problem or in facilitating price discrimination across product categories. These works (and ours) do not address the question of the optimal instrument to use, which can reflect other considerations, such as technological limitations (such as the inability to monitor the price and/or quantity of transactions) or coordination issues (participation-based fees may be infeasible when platforms face a chicken-and-egg problem to launch). For this line of inquiry, see Hagiu and Wright (2018).

In our general framework of platform governance, we have in mind decisions regarding quality control, entry regulation, as well as search and interface design. This brings our framework closer to recent studies on each of these specific topics. However, as described below, our research questions and the mechanisms driving our results significantly differ from each of these studies:

☐ **Quality control.** Jeon and Rochet (2010) analyzes how the quality standard decisions of an academic journal depend on whether it operates as an "open access" journal (charging nothing to readers) or a standard subscription-based journal. They show that the resulting quality standard is too high relative to welfare benchmarks if the journal charges readers for access, while the standard is too low if the journal is open access. Bouvard and Levy (2018) consider a certification intermediary that can invest in the quality of its certification technology, and they show the intermediary may invest too little in quality when its revenue relies on participation by low-quality firms. In contrast to these works, however, we consider a marketplace setting in which a platform intermediates trades between consumers and multiple competing price-setting sellers. We show that the incentive to manipulate on-platform competition provides another explanation for why platforms may set quality controls that are either too restrictive or too permissive.

☐ **Platform search diversion and obfuscation.** Hagiu and Jullien (2011) and White (2013) consider a platform that can garble the consumer search process in order to divert consumers towards the seller that generates a higher revenue for the platform. In doing so the platform trades off between earning a higher margin per consumer versus less consumer participation. However, such search diversion has no impact on the price-competition among sellers, which is the main trade-off in our framework. Casner (2019) analyze a platform's incentive to increase consumer search cost (i.e., to obfuscate search) in an on-platform consumer search environment based on the random sequential search model of Wolinsky (1986). He independently obtains one of the same findings — a platform with a proportional fee has an incentive to obfuscate search to sustain seller markups. However, Casner's analysis is restricted to an exogenously fixed proportional fee on sellers, whereas our framework endogenizes the level of the platform's fee and also considers other fee instruments under which the platform may have no incentive to obfuscate search. Moreover, our framework is readily applicable to study other non-random search environments, in particular, the price-directed search environment (e.g. Armstrong and Zhou, 2011; Armstrong, 2017; Choi et al., 2018), whereby the platform's incentive to obfuscate search can get reversed.

☐ **Entry regulation and variety provision.** Casadesus-Masanell and Halaburda (2010) provide a model of a two-sided platform connecting users with "applications" that themselves exhibit positive network externalities. Veiga (2018) considers a one-sided platform that can price-discriminate by segregating its users into different "sub-groups" that each exhibits positive network externalities. In these papers, the platform may find it profitable to restrict access in order to facilitate coordination among end-users at the level of applications or subgroups. In contrast, in our framework the platform restricts access (of sellers) to increase sellers' markup whenever its fee instrument aligns its interest with them.

2 Model setup

The environment consists of a continuum of unit-demand buyers, multiple sellers (can be finite or a continuum), and a platform that enables transactions between buyers and sellers. In what follows, we first present a general framework that is meant to encompass several different models of platform governance. We then illustrate the framework with three specific micro-foundations in Section 2.1.

Governance design. The platform chooses a governance design $a \in \Theta$, where $\Theta \subseteq \mathbb{R}$ is a compact (possibly discrete and finite) set. The design a affects the gross transactional value $V(a)$ that each buyer obtains from joining the platform and purchasing items from the sellers, where $V(a)$ is continuous for all $a \in \Theta$.[1] We normalize to zero the buyer's utility from the outside option. We interpret a as, among other interpretations, regulating entry, quality control, and design or technology choices. We allow $V(a)$ to be non-monotone, while noting that in most cases it is natural to think of $V(a)$ as an increasing function: admitting more sellers increases product variety; raising the quality standard increases expected product value; improving search quality increases buyers' reservation value. To highlight our main points in a simple fashion, we assume that the platform faces no fixed and marginal costs, including no cost of a higher (or lower) a.

Seller pricing. For each design a chosen by the platform, sellers engage in price competition to attract buyers. Sellers have a constant marginal cost $c > 0$. Suppose we ignore any platform fees at the moment. Without imposing any specific micro-foundation, we posit that the seller competition results in a symmetric equilibrium price

$$p = c + M(a) . \tag{1}$$

Here, $M(a) > 0$ is a markup function that captures the equilibrium markup that sellers earn. We assume that $M(a)$ is continuous for all $a \in \Theta$. With an arbitrary form of $M(a)$, the equilibrium price equation (1) is consistent with those arising from various micro-foundations with unit-demand consumers, e.g. the Perloff-Salop (1985) discrete choice model, the circular city model of Salop (1979), spokes model of Chen and Riordan (2007), the sequential search model of Wolinsky (1986) and Anderson and Renault (1999), and the price-directed search model of Choi et al. (2018), among others. The reduced-form formulation allows us to concisely capture the effect of platform governance on how the seller competition unfolds by specifying how $M(a)$ changes with a.

Buyers and volume of transactions. We let the total number of transactions (or

[1] Note $V(a)$ is a continuous function by definition whenever Θ is a discrete set.

aggregate demand) faced by the platform be

$$Q = Q\left(V\left(a\right) - p\right), \tag{2}$$

where Q is strictly increasing, continuously differentiable, and log-concave. For micro-foundations where the product market is not fully covered, Q represents the mass of consumers who forgo the outside option and purchase from one of the sellers. For micro-foundations with complete market coverage (where the aggregate demand, by default, is equal to the total mass of buyers), Q represents the mass of buyers that participate on the platform. To make Q elastic in the latter case, one can follow the approach of Edelman and Wright (2015a) by assuming that there is a continuum of buyers each of whom needs to incur a cost d to join the platform, and d is heterogenous across buyers so that only those with $d < V\left(a\right) - p$ would participate on the platform.

Platform fees. The platform levies a fee on sellers for each transaction, which can be a per-transaction fee τ, or it can be a proportional fee r (also known as a revenue sharing contract).[2] For notational brevity, we assume that the platform does not charge any transaction fee to buyers, which is without loss of generality due to the tax neutrality principle (Weyl and Farbinger, 2013).[3]

Under a per-transaction fee, the fee τ is essentially an additional marginal cost to sellers, so that the equilibrium price equation in (1) becomes

$$p = c + \tau + M\left(a\right).$$

Under a proportional fee, for each unit of sales revenue generated, a seller receives its share $1 - r$ while the platform keeps the remaining share r. For any given r, each seller's sales margin can be written as $(1 - r)\left(p - \frac{c}{1-r}\right)$. Ignoring the multiplicative factor, the seller sales margin is $p - \frac{c}{1-r} < p - c$ reflecting that a seller keeps only a share of its revenue but bears all of its costs of product, so that the seller acts as if its "effective" marginal cost is $\frac{c}{1-r}$. Hence, the equilibrium price equation in (1) becomes

$$p = \frac{c}{1-r} + M\left(a\right).$$

Discussion of modelling features. In (1), we implicitly assume that the $M\left(a\right)$ function is a primitive that does not depend on p. This implies a full pass-through of marginal cost to price, so that any transaction fee charged by the platform to sellers has

[2]The possibility of participation-based fee is explored in Section 4.2.

[3]In the marketplace environments we study, the standard principle of tax-neutrality - whereby sellers take into account the buyer-side fees when they set prices - implies that aggregate demand does not depend directly on the decomposition of platform fees between buyer fees and seller fees. Even if such neutrality does not hold, in many of the platform examples we have in mind, buyers do not face any fees, suggesting our focus purely on seller fees, is anyway a realistic assumption.

no impact on the level of markup that sellers earn. In Section 6, we discuss how this assumption can be relaxed by allowing the markup function to depend on p, in which case p becomes implicitly defined by $p = c + M(p, a)$. We also explore in the same section the possibility of generalizing the aggregate demand function Q in (2) to $Q = Q(V(a), p)$.

The assumption of costless governance design shuts down the well-known Spence (1975) distortion, that arises from the fact that a monopolist cares about the valuation of marginal users when increasing its product quality (or other product attributes) while the social planner cares about the valuation of average users. Given that an increase in product quality is always preferable by both marginal users and average users, when quality improvement is costless both the monopolist and the social planner will set the highest possible quality, and there will be no Spence distortion. Thus, the costless design assumption allows us to isolate the new form of welfare distortion that arises in this platform setting. Furthermore, in some applications such as website design choices, it may not be obvious how costs should vary with the level of a, so the costless design assumption is quite natural.

2.1 Micro-foundations

In this subsection, we provide three simple micro-foundations that fit the general framework presented above, whereby each example corresponds to a different aspect of platform governance design. To keep the exposition brief, we focus on showing how each example maps into the general framework, and relegate the detailed derivations to Section A of the Online Appendix. We will return to these examples in Section 5 when we discuss the implications from our analysis.

Example 1 Entry regulation and variety choice by platform (Perloff and Salop, 1985)

There is a continuum of unit-demand buyers and $n \geq 2$ ex-ante symmetric and horizontally-differentiated sellers. Let ϵ_i denote the random match value of a product i, which is identically and independently realized across buyers and products for $i = 1, ..., n$. Let F be the common cumulative distribution function (cdf) for all ϵ_i with log-concave density function f.

The platform chooses the number of sellers admitted to the platform, which we denote as $a \in \{2, ..., n\}$.[4] After observing a, a buyer chooses whether to incur a joining cost d to join the platform and learn the available products (specifically the match values and prices), where d is randomly drawn from a distribution with log-concave cdf G. Let Q denote the total number of participating buyers, then the demand faced by each admitted

[4]An alternative interpretation of entry regulation is the choice of the number of leads to show each buyer. Choosing more leads allows more sellers to enter each buyer's "consideration set", so that it is effectively equivalent to admitting more sellers to the platform.

seller i is $Q \times \int_0^\infty (1 - F(\epsilon - p + p_i))\, dF(\epsilon)^{a-1}$. Price competition among sellers lead to symmetric equilibrium price of the form $p = c + M(a)$, where the markup is

$$M(a) \equiv \frac{1}{a \int_0^\infty f(\epsilon)\, dF^{a-1}(\epsilon)}.$$

It follows from Anderson, de Palma, and Nesterov (1995) that $M(a)$ is strictly decreasing in a, reflecting that a higher number of sellers increases demand elasticity. Finally, a buyer joins the platform if the expected gain from doing so,

$$V(a) - p \equiv \mathbf{E}\left(\max_{i=1,\ldots,a}\{\epsilon_i\}\right) - p,$$

is greater than the joining cost. Thus, the total number of transactions is $Q = G(V(a) - p)$. Reflecting the usual logic of increased competition, an increase in a increases $V(a)$ and decreases $M(a)$ in this model.

Example 2 Quality control by platform Eliaz and Spielger (2011).

There is a continuum of unit-demand buyers and a continuum of sellers. Each seller i has quality $q_i \in [0,1]$, which is distributed according to the cdf H. When a buyer is matched with a seller of type q_i, with probability q_i the seller's product is non-defective and provides utility value $z + \epsilon_i$ where $\epsilon_i > 0$ is a consumer-product match component while z is a consumer-specific component invariant across products; With probability q_i the product is defective and provides only utility z. The values z and ϵ_i are identically and independently drawn across buyers and product, with distribution functions G and F and density functions g and f. All sellers face the same constant marginal cost c. Buyers know z before engaging in search. They search on the platform sequentially with perfect recall and they incur a search cost $s > 0$ each time they sample a seller. By sampling seller i, a buyer learns the product price p_i, the match value ϵ_i, and whether product i is defective, but the buyer never observes seller type q_i. The utility from a buyer's outside option of not purchasing yields zero utility.

The platform sets a minimum quality standard $a \in [0, \bar{a}] \subseteq [0,1]$, such that only sellers with quality $q_i \geq a$ are allowed to sell on the platform.[5] When buyers search, they only have access to the pool of sellers with $q_i \geq a$. Given that buyers do not observe each seller's quality but observe the platform's choice of minimum quality standard, they infer from a that the average quality of the seller pool is $E(q_i | q_i \geq a)$. Define a buyer's search

[5] This can be done by screening out low-quality sellers, or by a commitment to remove problematic listings. In practice, how strictly the platform's ranking algorithm penalizes listings with poor reviews will have a similar effect.

reservation value $V(a)$ implicitly as

$$\int_V^{\bar{\epsilon}} (\epsilon - V)\, dF(\epsilon) = \frac{s}{\mathbf{E}(q_i | q_i \geq a)}. \tag{3}$$

Notably, the "effective search cost" faced by consumers (right hand side of (3)) is decreasing in a. Following Weitzman (1979), the demand faced by a seller i is $Q \times \left(\frac{1-F(V(a)-p+p_i)}{1-F(V(a))}\right) q_i$, and the resulting symmetric equilibrium price is $p = c + M(a)$, where

$$M(a) \equiv \frac{1 - F(V(a))}{f(V(a))}.$$

It can be shown that if a consumer ever initiates search then she never chooses the outside option at any point during her search sequence, and she initiates search if and only if $z + V(a) - p \geq 0$. Therefore, the total number of transactions is $Q = 1 - G(p - V(a))$. Reflecting the standard effect of lowering search costs in a random consumer search setting (e.g. Wolinsky, 1986), an increase in a increases $V(a)$ and decreases $M(a)$ in this model.

Example 3 Design and on-platform search cost

Consider the following adapted version of price-directed search model of Choi et al. (2018).[6] There is a continuum of unit-demand buyers and $n \geq 2$ ex-ante symmetric and horizontally-differentiated sellers. All sellers face the same constant marginal cost c and set prices simultaneously. Buyers observe those prices and sequentially search for the best product with a per-search cost s. A buyer's net utility from buying a product i is $\epsilon_i + z_i - p_i$, where ϵ_i is the buyer's prior match value for product i that the buyer observes before inspecting the product, z_i is the residual match value that is revealed to the buyer only after inspecting the product, and p_i is the product price. The match values ϵ_i and z_i are random and independent of each other, and they are identically and independently drawn across buyers and products $i = 1, .., n$. Let F and H be the cdf of ϵ_i and z_i, with support over $[-\infty, \infty]$ and $[\underline{z}, \bar{z}]$ and log-concave density functions f and h respectively.

The platform can choose its search design $a \in [0, \bar{a}]$, where a higher a corresponds to a lower search cost for buyers (i.e., $s = s(a)$ is strictly decreasing in a). To join the platform a buyer must incur a joining cost d which is randomly drawn from a distribution with log-concave cdf G. To utilize Weitzman (1979)'s solution for consumer search, define reservation value z^* as

$$\int_{z^*}^{\bar{z}} (z_i - z^*)\, dH(z_i) = s(a).$$

Choi et al. (2018) provide an elegant characterization of each buyer's eventual purchase decision: for each i, define $w_i \equiv \epsilon_i + \min\{z_i, z^*\}$, then a buyer eventually purchases

[6] Another possible example is the random search model of Wolinsky (1986) and Anderson and Renault (1999).

product i if and only if $w_i - p_i > w_j - p_j$ for all $j \neq i$. If we let $\bar{H}$ denote the cdf of the new random variable $w_i \equiv \epsilon_i + \min\{z_i, z^*\}$ that is common for all i, and let $\bar{h}$ be its density function, then the symmetric equilibrium price in this environment can be derived as $p = c + M(a)$, where

$$M(a) \equiv \frac{1/n}{\int_{-\infty}^{\infty} \bar{h}(w|a)\, d\bar{H}^{n-1}(w|a)}.$$

Note that the distribution function $\bar{H}(w_i|a)$ is conditional on a because the variable w_i depends on z^*, which in turns is decreasing in the search cost $s(a)$. Finally, as a direct corollary of Choi et al.'s characterization, the expected surplus for a buyer from participating in the market is

$$V(a) - p \equiv \mathbf{E}\left(\max_{i=1,\ldots,n} \{w_i\} |a\right) - p.$$

Thus, in the equilibrium the mass of participating buyers of buyers joining the platform (and the total number of transactions) is $Q = G(V(a) - p)$. Reflecting the standard effect of lowering search costs in a price-directed search setting (e.g. Armstrong, 2017), an increase in a increases both $V(a)$ and $M(a)$ in this model.[7]

3 Baseline analysis: exogenous platform fees

Recall that the platform sets its governance design and its fee level simultaneously. To develop initial intuitions, in this section we analyze the general framework of Section 2 by assuming that the fee levels τ and r are exogenously fixed.[8] By doing so we shut down any distortion introduced by fee-setting decisions of the platform, which allows us to highlight distortions in the platform's governance designs. We first consider the case of per-transaction fees (Section 3.1), and then the case of proportional fees (Section 3.2).

3.1 Per-transaction fees

When the platform charges a per-transaction fee $\tau > 0$ to sellers, the equilibrium price that arises from seller competition is $p = c + \tau + M(a)$. Consider a profit-maximizing platform. Substituting for the seller equilibrium price, the platform's profit is $\tau Q(V(a) - M(a) - c - \tau)$. Without loss of generality we reformulate platform's problem

[7]See also the discussion in Section 5 for a detailed explanation of this logic and how the logic compares to the random search model of Wolinsky (1986).

[8]Alternatively, one may interpret the analysis in this section by considering the case that the fee levels are chosen before the governance design decision. For example, there may be cases where fees are adjusted less often than the governance choice, which is more flexible, for reasons outside the model.

as choosing the level of seller markup $m \in M(\Theta)$ directly whereby each m is associated with governance design $a(m) = M^{-1}(m)$ and seller gross surplus $v(m) \equiv V(a(m))$:[9]

$$\Pi(m) = \tau Q(v(m) - m - c - \tau).$$

We will apply the same reformulation technique throughout the main text.

We denote $m^p \equiv \arg\max_m \Pi$, where the superscript refers to "profit-maximization". It is immediate that

$$m^p = \arg\max_m \{v(m) - m\} \tag{4}$$

because the platform can enjoy a higher transaction volume (for any given τ) by increasing $v(m) - m$.

To identify the source of distortion in the profit-maximizing governance choice, we consider the welfare-maximization benchmark for comparison. Note that welfare-maximization is equivalent to a pure value-creation benchmark (Boudreau and Hagiu, 2009), that is, maximizing the total amount of economic value generated from user interactions on the platform. Total welfare is defined as the sum of joint industry profit (the platform and sellers) and buyers surplus. After substituting for the sellers' equilibrium price, total welfare (or total surplus generated) is written as

$$W(m) = (\tau + m) Q(v(m) - m - c - \tau) + \int_{-\infty}^{v(m)-m-c-\tau} Q(t)\, dt,$$

where the buyer (consumer) surplus is obtained by integrating the aggregate demand from $Q = 0$ up to the marginal demand.[10]

Define $m^w \equiv \arg\max_m W$, where the superscript refers to "welfare-maximization". Before proceeding, we note that this benchmark is welfare-maximizing only in a "partial" sense given that the platform fees are assumed to be fixed. In Section 4, where we endogenize the platform fee-setting decision, we consider the "second-best" welfare benchmark whereby there is a social planner that controls the governance design but does not control the fee decision of the platform.

With per-transaction fees, we claim that a profit-maximizing platform will choose a platform design that is associated with a higher level of seller competition (i.e. a lower markup) than the design that maximize total welfare, i.e. $m^p \leq m^w$. Note that if this

[9]The reformulation is valid whenever function $M(a)$ is a bijection. In cases where the bijectivity does not hold so that there are multiple a that solves $m = M(a)$, we can simply select a that corresponds to the highest $V(a)$ and the analysis will continue to hold.

[10]This specific reduced-form buyer surplus representation relies on our assumption that the buyers' outside option is normalized to zero. Our results would remain valid even if the outside option is some non-zero constant; we only require that it is not a function of the governance design.

were not the case and $m^p > m^w$, then

$$\begin{aligned} W(m^w) &\leq (\tau + m^w) Q(v(m^p) - m^p - c - \tau) + \int_{-\infty}^{v(m^p)-m^p-c-\tau} Q(t)\,dt \\ &< (\tau + m^p) Q(v(m^p) - m^p - c - \tau) + \int_{-\infty}^{v(m^p)-m^p-c-\tau} Q(t)\,dt \\ &= W(m^p), \end{aligned}$$

where the first inequality follows from the definition of m^p in (4), while the second inequality follows from the initial supposition that $m^p > m^w$. Obviously, $W(m^w) < W(m^p)$ contradicts the definition of m^w, hence:

Proposition 1 *(Exogenous per-transaction fees) Suppose the platform charges an exogenous per-transaction fee τ. The profit-maximizing governance design induces excessive on-platform competition (i.e. $m^p \leq m^w$).*

The economic intuition of Proposition 1 is easiest to understand if we assume that m is a continuous variable over some compact interval. We first note that welfare-maximization, whenever possible, calls for maximizing transactional value $v(m)$ and minimizing seller markup m using a single decision instrument, while the profit-maximizing platform seeks to do the same as well in an attempt to maximize transaction volume.

When $v(m)$ and m move in opposite directions, there is no trade-off in the choice of the optimal governance design from the perspectives of welfare-maximization and profit-maximization.[11] Decreasing m increases the transactional value and decreases the price set by sellers, so that it is welfare-improving and profit-improving to keep doing so until m^p and m^w reach the lowest possible level. There is no distortion in this case, as illustrated in the first panel of Figure 1 below.

In any region where $v(m)$ and m move in the same direction however, a trade-off arises: if one attempts to increase the transactional value $v(m)$, this would come with an implicit "cost" — the cost of increasing the seller markup and hence price. This implicit cost is greater from the perspective of a profit-maximizing platform because it focuses on transaction volume (which decreases if price increases). However, from a welfare perspective the said implicit cost is smaller because the loss in transaction volume (or output) from a higher price is partially offset by the corresponding gain in seller surplus (which increases if price increases). This implies that the welfare-maximizing design calls for a higher $v(m)$ compared to the profit-maximizing platform. As illustrated in the second panel of Figure 1, there is a downward distortion $m^p < m^w$ where the level of seller competition is too high and the associated transactional value for buyers is too low under profit-maximization.

[11] Note $v(m)$ and m move in opposite directions if and only if $V(a)$ and $M(a)$ move in opposite direction when a changes.

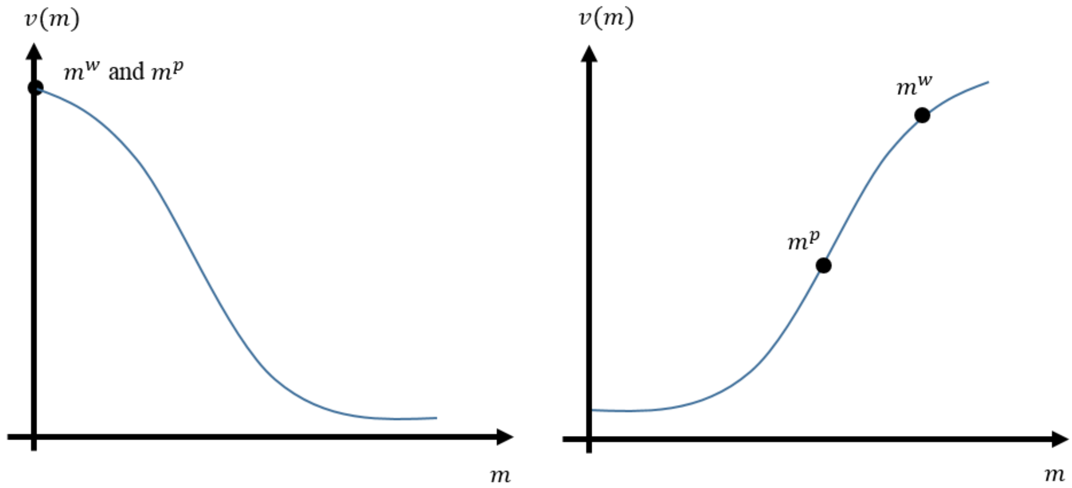

Figure 1: Distortions in governance design with per-transaction fee

3.2 Proportional fees

The logic behind Proposition 1, that the platform's profit (when choosing its governance design) is aligned with the maximization of transaction volume, can be reversed when the platform charges proportional fees instead. With a proportional fee $r > 0$, the equilibrium price that arises from seller competition is

$$p = \frac{c}{1-r} + m. \tag{5}$$

With a slight abuse of notation, we continue to denote platform's profit and total welfare as:

$$\begin{aligned} \Pi(m) &= rpQ(v(m) - p) \\ W(m) &= (p-c)\,Q(v(m)-p) + \int_{-\infty}^{v(m)-p} Q(t)\,dt, \end{aligned}$$

where p is given by (5).

After substituting in (5), the platform's profit can be written as

$$\frac{1}{r}\Pi(m) = \underbrace{mQ\left(v(m) - m - \frac{c}{1-r}\right)}_{\text{proportional to seller surplus}} + \frac{c}{1-r}\underbrace{Q\left(v(m) - m - \frac{c}{1-r}\right)}_{\text{volume of transactions}}. \tag{6}$$

Equation (6) essentially decomposes the platform's profit into two components. The first component in (6) is proportional to seller surplus that is given by

$$((1-r)p - c)\,Q(v(m) - p) = (1-r)\,mQ\left(v(m) - m - \frac{c}{1-r}\right).$$

This means that the platform's profit is fully aligned with the seller surplus if the second component of (6) is absent. The second component in (6) is proportional to the volume of transactions.

Therefore, (6) can be loosely interpreted as a weighted sum between seller surplus and the volume of transactions, in which the relative weight depends on c. This weighted-sum interpretation suggests that the direction of distortion in platform governance depends on the level of seller marginal cost. We confirm this intuition in the following proposition:

Proposition 2 *(Exogenous proportional fee) Suppose the platform charges an exogenous proportional fee r. There exist thresholds c^l and c^h, where $0 < c^l \leq c^h$, such that:*

1. *If $c < c^l$, the profit-maximizing governance design induces insufficient on-platform competition (i.e. $m^p \geq m^w$);*

2. *If $c > c^h$, the reverse is true (i.e. $m^p \leq m^w$).*

Proposition 2 essentially says that, when c is small such that the platform's profit under a proportional fee is skewed towards seller surplus, the profit-maximizing governance design induces a lower level of seller competition (i.e. a higher markup) than the welfare-maximizing design. Conversely, when c is large such that the platform's profit is skewed towards the volume of transactions, then it induces a higher level of seller competition (i.e. a lower markup) than the welfare benchmark instead.

To illustrate the intuition, we again suppose that m is a continuous variable over some compact interval. Consider first the case in which the sellers' marginal cost is very small so that $c \to 0$ (e.g. for sellers of digital products). Recall that welfare-maximization calls for maximizing transactional value for buyers $v(m)$ and minimizing seller markup m. Meanwhile, from (6) the profit-maximizing platform maximizes total seller surplus that can be extracted through the proportional fee, meaning that it seeks to maximize $v(m)$ and sustain a level of m that is not too low.

When $v(m)$ and m move in opposite directions, the welfare-maximizing design m^w is associated with the lowest possible level of markup. In contrast, the platform faces a trade-off because raising v requires lowering seller markup, so that the profit-maximizing design is generally associated with markup level $m^p \geq m^w$, i.e. less intense on-platform seller competition. So, the level of seller competition and $v(m)$ are both too low under profit-maximization, as illustrated in the first panel of Figure 2.

When $v(m)$ and m move in the same direction, the profit-maximizing platform faces no trade-off in this case because it can keep increasing m to increase both $v(m)$ and m as long as the resulting price is not above the price that maximizes joint industry profit. In contrast, there is a trade-off from the welfare perspective because increasing $v(m)$ comes at the cost of an increased price set by sellers. Therefore, welfare-maximization generally calls for $m^w \leq m^p$. Thus, the profit-maximizing governance design corresponds to a level of seller competition that is too low and a level of $v(m)$ that is too high, as illustrated in the second panel of Figure 2 below.

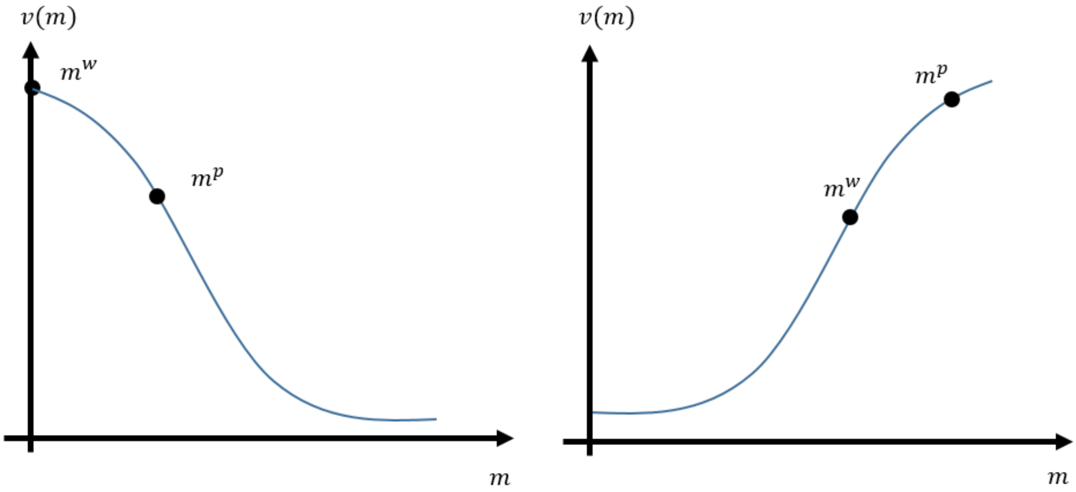

Figure 2: Distortions in governance design with proportional fees

As c increases above zero, the platform profit and the seller surplus begin to diverge as can be seen from (6). The divergence reflects that, under proportional fees, the platform internalizes sellers' revenue but does not internalize sellers' marginal cost. A higher c increasingly skews the platform's interest away from seller surplus. When c is high enough, the logic of Proposition 1 applies, whereby the profit-maximizing governance design is skewed towards maximizing the volume of transactions instead. Examples that fit this case include platforms that intermediate trades of high-value physical goods e.g. electronics or luxury products.

4 Analysis: endogenous platform fees

So far we have assumed that the fee level charged by the platform is exogenously fixed. We now relax this assumption by endogenizing the fee level. Throughout this section, the welfare benchmark we adopt is a "second-best" one, in which there is a social planner that fixes the governance design before the platform sets its fee so that the planner is constrained by the fee-setting response of the profit-maximizing platform. One interpretation is that the social planner regulates only the platform's governance while leaving its fees unregulated.[12] To facilitate the comparison with the welfare benchmark, in the derivation of the profit-maximizing design it is useful to think of the platform as choosing its governance design first and then choosing its fees (given the platform does not obtain

[12] In Section B of the Online Appendix, we consider yet another welfare benchmark where the social planner regulates both the platform governance design and the fee level. We show that all the qualitative insights from this section continue to hold.

any new information in between these two decisions, this reordering does not affect the formal analysis).

4.1 Transaction-based fees

Following Section 3.1, the platform's profit function is given by $\tau Q(v(m) - m - c - \tau)$, where the per-transaction fee τ is now endogenous. Given Q is log-concave, for each given m the profit-maximizing fee $\tau^p = \tau^p(m)$ is implicitly defined by first-order condition

$$\tau^p = \frac{Q(v(m) - m - c - \tau^p)}{Q'(v(m) - m - c - \tau^p)}. \tag{7}$$

Given this, we can write down the platform's profit function as an indirect function of m:

$$\tilde{\Pi}(m) = \tau^p(m) Q(v(m) - m - c - \tau^p(m))$$

To obtain the profit-maximizing governance design, by the envelope theorem we can ignore the indirect effect of m on τ^p so $m^p \equiv \arg\max \tilde{\Pi} = \arg\max \{v(m) - m\}$. Compared to the baseline case of exogenous transaction fees, maximizing $v - m$ has a two-fold effect here. First, it allows the platform to raise the number of transactions Q; second, it allows the platform to reoptimize by increasing its fee, which leads to an even higher profit. The latter point can be seen from (7), whereby Q being log-concave means that τ^p is increasing in $v(m) - m$ and it is maximized at $m = m^p$.

Given the endogenous transaction fee, the welfare function is given by

$$\tilde{W}(m) = (\tau^p(m) + m) Q(v(m) - m - c - \tau^p(m)) + \int_{-\infty}^{v(m)-m-c-\tau^p(m)} Q(t)\, dt. \tag{8}$$

We denote $m^{sb} \equiv \arg\max \tilde{W}(m)$, where the superscript refers to "second-best". Compared to the welfare benchmark in the baseline case, here the social planner needs to take into account the possibility that the platform may increase its fee in response to changes in the governance design.

Proposition 3 *(Endogenous per-transaction fees) Suppose a social planner can control the platform's governance design, but cannot control the per-transaction fee τ set by the platform. Then, the planner prefers a design that is associated with less on-platform competition than the platform (i.e. $m^{sb} \geq m^p$).*

Proposition 3 implies that the insights from Proposition 1 continue to hold when we endogenize the setting of the platform's fee. The key step in the analysis comes from the fact that τ^p features an incomplete pass-through, i.e. $\frac{d\tau^p}{d(v-m)} \in (0,1)$, so that the profit-maximizing governance design m^p also maximizes the transaction volume

$Q(v(m) - m - c - \tau^p(m))$. The result then follows from the same logic that establishes Proposition 1: the platform fails to internalize seller surplus and so it is unwilling to set a design that corresponds to a high seller markup even when doing so increases the transactional value generated.

Suppose instead the platform sets a proportional fee $r \in [0, 1]$ so that its profit function is given by $r\left(m + \frac{c}{1-r}\right) Q\left(v(m) - m - \frac{c}{1-r}\right)$. For each given m, setting $r = 0$ and $r = 1$ are obviously sub-optimal as they result in a zero profit for the platform. Therefore, the platform necessarily sets some $r^p(m) \in (0, 1)$ implicitly pinned down by first-order condition $d\Pi/dr = 0$, or

$$r^p = \frac{Q\left(v(m) - m - \frac{c}{1-r^p}\right)}{Q'\left(v(m) - m - \frac{c}{1-r^p}\right)} \left(\frac{(1-r^p)^2}{c} + \frac{r^p}{m + \frac{c}{1-r}}\right). \tag{9}$$

One can verify that the right hand side of (9) is decreasing in r^p, so that (9) has a unique solution. Given this, the platform's profit function becomes

$$\tilde{\Pi}(m) = r^p(m)\left(m + \frac{c}{1 - r^p(m)}\right) Q\left(v(m) - m - \frac{c}{1 - r^p(m)}\right). \tag{10}$$

The envelope theorem allows us to ignore the indirect effect of m on r^p, so that the profit-maximizing governance design $m^p \equiv \arg\max \tilde{\Pi}(m)$ here reflects the same underlying trade-off as its counterpart in Section 3.2. That is, the platform's profit function can be loosely interpreted as a weighted sum between seller surplus and transaction volume, in which the relative weight depends on c.

Denote the welfare function as

$$\tilde{W}(m) = \left(m + \frac{r^p(m)}{1 - r^p(m)}c\right) Q\left(v(m) - m - \frac{c}{1 - r^p(m)}\right) + \int_{-\infty}^{v(m)-m-\frac{c}{1-r^p(m)}} Q(t)\,dt, \tag{11}$$

and $m^{sb} \equiv \arg\max \tilde{W}(m)$. Comparing m^p and m^{sb} yields the following result that is analogous to Proposition 2.

Proposition 4 *(Endogenous proportional fee) Suppose a social planner can control the platform's governance design, but cannot control the proportional fee r set by the platform. There exist a threshold $c^h > 0$ such that*

1. *If $c \to 0$, the planner prefers a design that is associated with more on-platform competition than the platform (i.e. $m^{sb} \leq m^p$).*

2. *If $c > c^h$, the reverse is true (i.e. $m^{sb} \geq m^p$).*

4.2 Participation fees and two-part tariffs

Thus far, our analysis has focused on platform fees that are transaction-based. However, in some contexts a platform may find it hard to implement transaction-based fees, e.g., price comparison websites in housing or rental markets cannot always reliably monitor transactions because deals are typically conducted outside the platform. Instead of transaction-based fees, these platforms typically charge sellers an up-front participation fee in the form of listing fees. Notable examples include rental market websites such as Rightmove and Zoopla in the United Kingdom, or Immobilienscout24 and Immowelt in Germany. To analyze this arrangement, suppose that the platform charges a participation fee T_S on sellers. It is easy to see that the platform does best by choosing a governance design that maximizes total seller surplus and then sets T_S to fully extract the total seller surplus. The platform's profit equals seller surplus

$$\tilde{\Pi}(m) = mQ(v(m) - m - c). \tag{12}$$

Similar to the logic of Proposition 4 (with small c), the profit-maximizing platform's choice of governance design induces a lower level of seller competition than the level that the planner desires, i.e. $m^p \geq m^{sb}$. Formally:

Corollary 1 *(Participation fees) Suppose a social planner can control the platform's governance design, but cannot control the participation fee set by the platform. Then, the planner prefers a design that is associated with more on-platform competition than the platform (i.e. $m^{sb} \leq m^p$).*

Finally, we consider two-part tariffs, whereby the platform charges sellers a participation fee T_S and either (i) a per-transaction fee $\tau \geq 0$; or (ii) a proportional fee $r \geq 0$. We rule out the possibility of sellers being charged a negative transaction fee as it requires the platform to subsidize transactions. Such a subsidy is rare in practice, and it is difficult to implement because each seller can potentially fabricate transactions by fraudulently purchasing from itself in order to claim the transaction subsidies.

Under a two-part tariff, the profit-maximizing platform sets T_S to fully extract the total seller surplus. The resulting profit function is the sum of participation fees and the transaction fees collected, that is,

$$(\tau + m) Q(v(m) - m - \tau - c) \quad \text{under a per-transaction fee}$$

or

$$\left(\frac{c}{1-r} + m - c\right) Q\left(v(m) - m - \frac{c}{1-r}\right) \quad \text{under a proportional fee}$$

To proceed, denote $m^* = \arg\max_{m \in M(\Theta)} v(m)$. Then, define $p^*(m)$ as the monopoly price that maximizes the joint industry profit, i.e. it solves $p^* = c + \frac{Q(v(m)-p^*)}{Q'(v(m)-p^*)}$.

We first note that the platform uses a participation fee to extract all seller surplus so that its profit function is proportional to the joint industry profit. When c is low ($m^* < p^*(m^*) - c$), the platform sets its governance design at m^* to maximize gross transactional surplus, and then adjusts τ or r accordingly to implement the corresponding monopoly price $p^*(m^*)$. When c is high ($m^* \geq p^*(m^*) - c$), inducing the monopoly price would require a subsidy, which is ruled out by the non-negative constraint on fees. In this case, the platform sets $\tau = 0$ or $r = 0$ such that its profit function becomes (12) as in a pure participation fee model, meaning that the profit-maximizing governance design maximizes seller surplus.

Comparing the profit-maximizing governance design with what the social planner would choose, we have:

Proposition 5 *(Two-part tariff) Suppose a social planner can control the platform's governance design, but cannot control the two-part tariff set by the platform. If m^* is unique, then the planner prefers a design that is associated with more on-platform competition than the platform (i.e. $m^{sb} \leq m^p$).*

Even though Corollary 1 and Proposition 5 have utilized the fact that the platform fully extracts all seller surplus through the participation fee, these results continue to hold even if sellers face some uncertain fixed cost C, such that the platform does not fully extract all seller surplus. To see this, consider a simple example in which there are two possible realizations of seller fixed cost C_l and C_h, where $C_h > C_l$. Each realization occurs with probabilities λ and $1 - \lambda$ respectively, and the actual realization of fixed cost is not observed by the platform. In case the platform is better off setting the participation fee at $mQ(v(m) - p) - C_l$ in which sellers participate regardless of the realized cost, then the platform does not extract all the surplus, and the only amendment to the analysis is an extra fixed cost term in the platform's profit expression, which does not affect the determination of the optimal governance design.

If we endogenize the platform's choice of fee instrument, then in our baseline setting, the two-part tariff obviously dominates among the fee instruments analyzed. However, in reality there are several reasons why a two-part tariff may not be optimal. As mentioned earlier, if the platform cannot reliably monitor transactions, then a transaction-based fee component may not be feasible. On the other hand, a participation-based fee component may give rise to the possibility of a chicken-and-egg coordination problem and leads to a no-participation outcome. Another possible reason for not using participation fees is that the sellers may face liquidity constraints in which they could not pay up-front fees before getting their products sold.

4.3 Summary

The analysis in this section highlights that the platform's incentives in choosing its governance design are strongly tied to its fee instrument.

Fee instrument:	Per-transaction fees / Proportional fees (high seller cost)	Seller participation fees / Proportional fees (low seller cost) / Two-part tariffs
Platform's incentive in setting governance design:	Volume-aligned: to *intensify* on-platform competition	Seller-aligned: to *relax* on-platform competition

Table 1: Summary

To summarize the main insights, we categorize each of the analyzed platform fee instruments according to the direction of the welfare distortion in platform governance design. We group per-transaction fees and proportional fees (when sellers' marginal cost is high) as *volume-aligned* fee instruments. Under these fee instruments, we find that the profit-maximizing platform prefers to sets its governance design to increase the volume of transactions, and its choice of governance design is distorted towards intensifying seller competition.

Likewise, we group seller participation fee, proportional fee (when sellers' marginal cost is low), and two-part tariffs charged to sellers as *seller-aligned* fee instruments. Under these fee instruments, we find that the profit-maximizing platform's choice of governance design is skewed towards increasing the seller surplus that it can extract, and its choice of governance design tends to induce too little seller competition.

Whether these distortions ultimately result in the platform choosing an excessive or insufficient level of the governance design depends on the details of how the governance design affects the transactional value for buyers and the extent of seller competition, which we explore in Section 5.

5 Applications and implications

We apply the general insights developed in Sections 3 and 4 to three different types of platform governance designs, corresponding to Examples 1-3 in Section 2.

□ **Entry regulation and variety choice by platform.** In the entry regulation model of Perloff and Salop (1985) presented in Example 1, admitting more sellers has two effects: (i) increased product variety; and (ii) reduced sellers' markup. Both effects increase welfare and aggregate demand, so that with volume-aligned fee instruments, both welfare-maximization and profit-maximization call for admitting the highest possible number of sellers, n. In contrast, with seller-aligned fee instruments, intensifying seller competition reduces the platform's revenue. Therefore, if the markup-reducing effect dominates, the platform will admit strictly less than n sellers.

To illustrate the implications, consider the example of shopping malls. A shopping mall typically charges its tenants a two-part tariff consisting of rent payments (participation fees) and proportional fees on sales revenue of tenants. Proposition 5 thus suggests that a shopping mall tends to restrict too much the number of competing tenants (in each given product/service category) from a welfare perspective. One way such restrictions can be achieved is to impose a high selection criterion for tenants. For example Roppongi Hill, a prestigious "mini-city" shopping mall located in Tokyo, is well known for its demanding quality requirements on its tenants (Boudreau and Hagiu, 2009). Once taken in, tenants are encouraged to periodically renew the designs of their stores, and the mall actively replaces underperforming stores or those found not to fit the mall environment. Another way is to simply offer a contract that grants exclusivity to a single tenant by committing not to admit the tenant's competitors into the mall, as documented by Ater (2015). In this sense, one can also interpret the decision of whether to grant category exclusivity as an additional non-price governance decision by platforms (Karle at al., 2019)

Another relevant example is video game platforms, e.g. Sony PS4, Microsoft Xbox One, and Nintendo. These platforms price their game consoles to buyers at approximately cost, generating most of the revenue through charging game developers/publishers a two-part tariff consisting of developer kit fees (i.e. participation fees) and fixed per-game licensing payment (i.e. constant per-transaction fees). Again, Proposition 5 predicts that these platforms tend to restrict the number of competing video game titles too much in order to sustain the profit of major game developers. Consistent with this prediction, Sony, Microsoft, and Nintendo indeed restrict access to a selected set of game developers and exclude many others, as documented by Evans et al. (2008).

☐ **Quality control by platform.** In online markets, the prevalence of information asymmetry between buyers and sellers means that platforms need to carefully regulate the quality of the listed sellers. In the quality control model of Eliaz and Spielger (2011) presented in Example 2, a search pool with a higher expected quality is analogous to a lower "effective search cost" for buyers. This reflects that each buyer searches less and consequently incurs a lower total expected search cost of $s/\mathbf{E}\left(q_i|q_i \geq a\right)$ before reaching a non-defective (positive-valued) match. Given that $\mathbf{E}\left(q_i|q_i \geq a\right)$ increases with a, a higher quality standard set by the platform is analogous to reducing effective search cost of buyers. Therefore, raising the quality standard has two effects: (i) increased buyer reservation value; and (ii) reduced seller markup.[13] Both effects are similar to previous

[13]Consistent with our formulation, Hui et al. (2019) provide empirical evidence showing that a more

example. If the markup-reducing effect dominates, with seller-aligned fee instruments, a platform's choice of quality standard will be strictly less than the welfare-maximizing one.

To illustrate the implications, consider the example of PC games distribution platforms such as Steam. With the extensive online user reviews available, prospective consumers can learn the performance of each game if they carefully go through the review system, which fits our model of observable product defects. Steam charges game developers a flat proportional fee 30% for each transaction on the platform, while developers typically have a marginal cost that is close to zero. Therefore, Proposition 4 predicts that the platform does not have an incentive to set a very strict quality control. Reportedly, Steam has shown reluctance in imposing simple low cost measures that could significantly increase the average quality of product pool on offer.[14] Our result suggests that an explanation for the reluctance to engage in stricter quality control could be its attempt to allow PC developers to maintain higher prices and revenues, which the platform can extract through its proportional fee.

We note that even though our analysis has focused on the role of a high quality standard in facilitating search (which intensifies seller competition), in some contexts a high quality standard can plausibly generate countervailing effects that relax seller competition instead. For instance, if the number of sellers is finite, excluding low-quality sellers reduces the total number of sellers akin to entry restriction, which relaxes seller competition. Given that entry regulation has been discussed in Example 1, for simplicity we have chosen to shut down this channel by assuming that there is a continuum of sellers. If we amend Example 2 by assuming a finite number of sellers, the overall effect of a high quality standard on the extent of seller competition generally depends on whether the search-facilitating effect or the entry-restriction effect dominates.

□ **On-platform search friction.** An online platform often makes design decisions that influence the ease of buyer search on the platform. These decisions may involve designing its search and recommendation algorithm, organizing information displayed on the user interface, or using its recommendation to divert search. Each of these decisions ultimately affects the search cost incurred by buyers to inspect product attributes. When platforms can costlessly manipulate buyers' search cost, the question is: would the platform finds it profitable to obfuscate search, that is, not minimizing buyers' search cost?

A natural starting point to analyze search cost manipulations is Wolinsky's (1986) random search model, where we can think of the platform governance design a as the search quality, whereby a high value of a corresponds to a low buyer search cost. In

stringent quality certification policy on ebay has increased the extent of on-platform seller competition while at the same time increasing the average quality of the seller pool.

[14]See https://www.rockpapershotgun.com/2017/02/14/steam-curation-user-reviews- xes/

Wolinsky's model, lowering search cost has two effects: (i) increased reservation value, and (ii) intensified seller competition as buyer demand becomes more elastic. Obviously, welfare-maximization calls for the highest possible search quality (lowest possible search cost). As for the profit-maximizing platform, the results in Sections 3 and 4 imply that the platform has no incentive to obfuscate buyer search if its fee instrument is volume-aligned, consistent with the conventional wisdom (Dinerstein et al., 2019) that platforms want to limit search frictions and provide buyers with transparent and low prices. However, the results also imply that a platform would want to obfuscate search if its fee instrument is seller-aligned instead, consistent with Hagiu and Jullien's (2011) point that platforms do not always want to eliminate search frictions.[15] Thus, our framework offers a reconciliation between Hagiu and Jullien (2011)'s point and the conventional wisdom by showing that the platform's incentive to reduce search frictions can go in either direction depending on the fee instrument employed by the platform.

Departing from Wolinsky's random search model, another interesting setting to analyze search cost manipulations is to allow buyer search to be price-directed, e.g. the model of Choi et al. (2018) in Example 3.[16] In a price-directed search setting, buyers can observe prices before sampling for product match values. This feature is particularly relevant in the context of price-comparisons websites, whereby a buyer first looks at a list of product-price offers before clicking on offers that she wants to spend time investigating further. In this model, raising the search quality has two effects: (i) increased gross buyer participation surplus; and (ii) increased seller markups. To understand why lowering search costs actually relaxes seller competition, consider the following intuition. A higher search cost means that buyers become less likely to visit another seller after having visited the first seller. This makes it worthwhile for each seller to set a low price and attract buyers to visit it first (recall that buyers' search sequence is influenced by the prices they observe). Due to this mechanism, a higher search cost makes demand more price-elastic in a price-directed search environment as opposed to the random search environment of Wolinsky (1986).[17]

Notably, given that raising search quality (lower search costs) can increase seller markups, the welfare-maximizing search quality optimally balances between higher search quality and avoiding high seller markups. Meanwhile, a profit-maximizing platform with volume-aligned fee instruments prefers an even lower seller markup than a social planner

[15] In the current paper the exact mechanism for this result differs from those in Hagiu and Jullien (2011). In our setup, search diversion or obfuscation relaxes seller competition, increases seller pricing, and increases the revenue for the intermediary (through a proportional fee) from each consumer. Hagiu and Jullien shut down this channel by assuming that the sellers are either (i) independent or (ii) interdependent but unable to adjust their prices in response to any diversion. In their model, the intermediary obtains revenue for each store visit by consumers, and search diversion increases the number of store visit for each consumer that goes to the intermediary.

[16] See also Armstrong and Zhou (2011) and Armstrong (2017).

[17] In the random search model, prices are unobservable before search so that a higher search cost makes demand less price-elastic instead.

hence it may choose a search quality that is too low, while a platform with seller-aligned fee instruments may choose a search quality that is too high. The results thus suggest that price comparison websites, which typically charge sellers a listing fee for displaying their offers on the platforms, have a particularly strong incentive to continuously improve the layout of information in order to facilitate search on the platforms.

6 Extension: Incomplete pass-through and market coverage

Our baseline analysis has focused on micro-foundations with unit-demand consumers and complete market coverage, which implies a full pass-through of marginal cost to price. An alternative formulation is to consider micro-foundations with incomplete market coverage, which we consider in this section. To keep the exposition brief, we relegate formal derivations and analysis to Section C of the Online Appendix.

Suppose that in the absence of platform fees, seller competition results in a symmetric equilibrium price defined implicitly by

$$p = c + M(p, a),$$

which is analogous to the price equation (1). We assume that the markup function $M(p, a)$ is differentiable and decreasing in its first argument so that $p = p(a)$ is uniquely defined for each given c and a. This assumption implies an incomplete pass-through of marginal cost to price, so that a one unit increase in marginal cost will lead to a less than one unit increase in the equilibrium price. The aggregate demand remains denoted as $Q(V(a) - p)$.

By allowing the markup term to depend on price, this demand formulation is consistent with the micro-foundations of incomplete market coverage. To make things concrete, consider the following amended version of the representative consumer model by Shubik and Leviatan (1980):[18]

Example 4 Entry regulation and variety choice in Shubik and Leviatan (1980)

Suppose there is a finite number of $n \geq 2$ ex-ante symmetric unit-product sellers. Let $a \in \{1, ..., n\}$ represents the number of sellers admitted to the platform. A representative consumer, having access to the admitted sellers, has utility

$$U = V \sum_{i=1}^{a} Q_i - \frac{a}{2}\left((1-\gamma)\sum_{i=1}^{a} Q_i^2 + \frac{\gamma}{a}\left(\sum_{i=1}^{a} Q_i\right)^2\right) + Y - \sum_{i=1}^{a} p_i Q_i.$$

[18] Another example that fits is the Singh-Vives-Hackner (2000) model.

Here, V is the direct utility from product consumption; $\gamma \in (0,1)$ is a measure of product differentiation; Q_i and p_i are the quantity of product i consumed and its price; and $Y - \sum_{i=1}^{a} p_i Q_i$ is the residual income of the consumer after the total expenditure on the products. We extend Shubik and Leviatan's model by allowing $V = V(a)$ to be increasing in a, so that the consumer's utility is increasing in product variety available on the platform. We can then solve for the consumer's demand for each product i as

$$Q_i = \frac{1}{a}\left(V(a) - \frac{p_i}{1-\gamma} + \frac{\gamma}{1-\gamma}\sum_{i=1}^{a}\frac{p_i}{a}\right).$$

Each seller faces a constant marginal cost c and solves $\max_{p_i}(p_i - c)\,Q_i$, implying the symmetric equilibrium price $p = c + M(p,a)$, where

$$M(p,a) = \left(\frac{1-\gamma}{1-\gamma/a}\right)(V(a) - p).$$

The aggregate demand faced by platform is $Q = \sum_i^a Q_i = V(a) - p$.

The main insights from Section 3, namely, Propositions 1 and 2, remain unaffected in this more general environment. The main step to generalize the results is to define the "composite" markup function

$$\bar{M}(a) = M(p(a), a),$$

which allows us to capture how the markup term changes with a after taking into account the indirect effect through price. It is easily verified that $\bar{M}(a)$ is increasing in a if and only if the original markup function $M(p,a)$ is increasing in its second argument. Therefore, the analysis in Section 3 continue to hold after replacing the markup functions with $\bar{M}(a)$.

Extending further, one can also generalize the functional form of the aggregate demand function by letting $Q = Q(V(a), p)$, where Q is strictly increasing in the transactional value for buyers V and strictly decreasing in the equilibrium product price p. This alternative formulation uncouples the one-to-one relationship between the total transaction volume $Q(V(a), p)$ with the buyer surplus $\int_p^\infty Q(V(a), t)\,dt$ (both of which are increasing in $V(a) - p$ in our baseline setting). Nonetheless, the same monotone comparative static argument that establishes Propositions 1 and 2.2 remains applicable. If in addition we assume that the aggregate demand rate of substitution $\Psi \equiv \frac{\partial Q(V,p)/\partial p}{\partial Q(V,p)/\partial V} \leq 0$ is (weakly) decreasing in p, then 2.1 also continues to hold.

7 Conclusion

An important distinction between a platform (marketplace) business and a traditional retailer is that the platform hosts groups of sellers that make independent pricing decisions, whereas the retailer sells and prices all its products directly. Without the ability to determine prices directly, the platform may want to use its governance designs to influence prices indirectly through the effect of its governance on the extent of seller competition. The current paper systematically demonstrates how this motive to manage seller competition influences a platform's choice of governance design and drives the difference between the profit-maximizing and the socially-optimal governance designs.

As summarized in Table 1, the direction in which the platform uses its governance designs to manage seller competition crucially depends on the revenue-generating model it employs. Given that prices enter transaction volume and seller surplus in opposite directions, each platform business can be seen as positioning itself on a continuum of revenue-generating models. At one end, there is a pure volume-aligned model in which the platform prefers governance designs that induce more intense seller competition than the socially optimal design. At the other end, there is a pure seller-aligned model in which the platform prefers governance designs that induce less intense seller competition than the socially optimal design.

Naturally, there are other factors that distorts a platform's choice of governance design that our framework did not capture. For example, if the cost governance is taken into account then the classic Spence (1975) distortion arises. Alternatively, if the platform operator is vertically integrated with some of the on-platform sellers, it may engage in abusive self-preferencing design decisions, e.g. shunning away rival sellers. Last but not least, if sellers are asymmetric, the platform distort its search ranking mechanism to "steer" buyers to low-quality sellers if such sellers offer higher per-transaction revenue to the platform (Teh and Wright, 2019). To this end, the current paper shows that distortion can arise even in the absence of governance cost, vertical integration, and asymmetry among sellers.

To extend our framework, an obvious direction is to investigate how competition between platforms affects platform governance design. With two rival platforms, it is natural for each seller to join both intermediaries and for each buyer to join only one. This leads to a competitive bottleneck equilibrium similar to that analyzed in Armstrong and Wright (2007). Inter-platform competition implies a transaction volume that is more elastic (with respect to the net utility offered to buyers) than the case of a monopoly platform, which induces the platforms to adjust their governance design towards inducing more on-platform competition, so as to achieve a more competitive price on their respective platforms. Following this intuition, we expect that the introduction of inter-platform competition shifts each platform business to be more volume-aligned.

In our model, we focus on the situation in which platforms charge fees only to sellers. This is a common practice among most online marketplaces.[19] An important reason is that buyers are often uncertain about whether or not they want to buy a product, and they first inform themselves on the platform about characteristics of the available products. Thus, charging a participation fee on buyers will deter many buyers. Alternatively, buyer participation fee may be infeasible if a platform cannot monitor participation decision by buyers. Nonetheless, in contexts where buyer participation fee is feasible, a natural question is how does the platform governance design changes when it charges participation fees on both sides. Belleflamme and Peitz (2018) consider a two-sided participation fee model, and focus on exploring how the platform fee structure is affected by the extent of on-platform seller competition. They also provide an informal discussion on how the platform profit may be affected when platform can make non-price decisions that affect the extent of the platform competition.

8 Appendix

8.1 Proofs

Proof. (**Proposition 1**)**.** The result follows from the proof in the main text. ■

Proof. (**Proposition 2**)**.** Let $Q' > 0$ denote the derivative of Q. Given that Q is strictly increasing, the inverse function Q^{-1} exists and is a strictly increasing function. Hence, we have the following identity:

$$v(m) - p = Q^{-1}\left(\frac{\Pi(m)}{rp}\right) = Q^{-1}\left(\frac{\Pi(m)/r}{\frac{c}{1-r} + m}\right). \tag{13}$$

Substituting identity (13) and the price equation $p = \frac{c}{1-r} + m$, the welfare function becomes

$$W(m) = \left(1 - \frac{c}{\frac{c}{1-r} + m}\right)\frac{\Pi(m)}{r} + \int_{-\infty}^{Q^{-1}\left(\frac{\Pi(m)/r}{\frac{c}{1-r}+m}\right)} Q(t)\,dt. \tag{14}$$

We first identify conditions under which expression (14) decreases or increases with the term m, holding $\Pi(m)$ constant. Since W is differentiable with respect to m (given Q is continuously differentiable), the corresponding derivative is

$$\frac{dW}{dm}|_{\Pi} = \frac{\Pi(m)/r}{\left(\frac{c}{1-r} + m\right)^2}\left[c - \frac{Q\left(v(m) - \frac{c}{1-r} - m\right)}{Q'\left(v(m) - \frac{c}{1-r} - m\right)}\right],$$

where we uses (13). Let

$$m^h \equiv \arg\max_m \{v(m) - m\} \text{ and } m^l \equiv \arg\min_m \{v(m) - m\},$$

both of which are well-defined by compactness of the domain and continuity of $v(m) - m$. Then:

[19]See, e.g. Nocke, Peitz and Stahl (2007) and Karle, Peitz, and Reisinger (forthcoming) for two-sided platform models that similarly focus on seller-side fees.

- Let c^l be the unique solution to

$$c-\frac{Q\left(v\left(m^l\right)-m^l-\frac{c}{1-r}\right)}{Q'\left(v\left(m^l\right)-m^l-\frac{c}{1-r}\right)}=0. \tag{15}$$

The existence and uniqueness of c^l follows from the intermediate value theorem given log-concavity of Q (so that Q/Q' is increasing). By construction, for all $c<c^l$ and for all $m\in M(\Theta)$, we have

$$c-\frac{Q\left(v(m)-m-\frac{c}{1-r}\right)}{Q'\left(v(m)-m-\frac{c}{1-r}\right)}\le c-\frac{Q\left(v\left(m^l\right)-m^l-\frac{c}{1-r}\right)}{Q'\left(v\left(m^l\right)-m^l-\frac{c}{1-r}\right)}<c^l-\frac{Q\left(v\left(m^l\right)-m^l-\frac{c^l}{1-r}\right)}{Q'\left(v\left(m^l\right)-m^l-\frac{c^l}{1-r}\right)}=0,$$

where the inequalities are due to log-concavity of Q and the definition of c^l. It follows that $\frac{dW}{dm}|_\Pi<0$ for all $c\le c^l$ and for all $m\in M(\Theta)$

- Let c^h be the unique solution to

$$c-\frac{Q\left(v\left(m^h\right)-m^h-c\right)}{Q'\left(v\left(m^h\right)-m^h-c\right)}=0. \tag{16}$$

The existence and uniqueness of c^h follow from the same reasons above. By construction, for all $c\ge c^h$ and for all $m\in M(\Theta)$, we have

$$c-\frac{Q\left(v(m)-m-\frac{c}{1-r}\right)}{Q'\left(v(m)-m-\frac{c}{1-r}\right)}\ge c-\frac{Q\left(v\left(m^h\right)-m^h-c\right)}{Q'\left(v\left(m^h\right)-m^h-c\right)}>c^h-\frac{Q\left(v\left(m^h\right)-m^h-c^h\right)}{Q'\left(v\left(m^h\right)-m^h-c^h\right)}=0.$$

It follows that $\frac{dW}{dm}|_\Pi>0$ for all $c>c^h$ and for all $m\in M(\Theta)$.

Consider first the case in which $c\le c^l$ so that (14) decreases with m. To show $m^p\ge m^w$, by contradiction suppose instead the reverse is true so that $m^p<m^w$. Then, from (14),

$$\begin{aligned}W\left(m^w\right) &< \left(1-\frac{c}{\frac{c}{1-r}+m^p}\right)\frac{\Pi\left(m^w\right)}{r}+\int_{-\infty}^{Q^{-1}\left(\frac{\Pi\left(m^w\right)/r}{\frac{c}{1-r}+m^p}\right)}Q(t)\,dt\\ &\le \left(1-\frac{c}{\frac{c}{1-r}+m^p}\right)\frac{\Pi\left(m^p\right)}{r}+\int_{-\infty}^{Q^{-1}\left(\frac{\Pi\left(m^p\right)/r}{\frac{c}{1-r}+m^p}\right)}Q(t)\,dt=W\left(m^p\right),\end{aligned}$$

where the first inequality follows from $\frac{dW}{dm}|_\Pi<0$ and the initial supposition $m^p<m^w$, while the second inequality from the definition of $m^p=\arg\max_m \Pi(a)$. So $W(m^w)<W(m^p)$, which contradicts the definition of m^w. Thus, we must have $m^p\ge m^w$ when $c<c^l$.

Consider $c\ge c^h$. To show $m^p\le m^w$, by contradiction suppose instead the reverse is true so that $m^p>m^w$. Then, from (14),

$$\begin{aligned}W\left(m^w\right) &< \left(1-\frac{c}{\frac{c}{1-r}+m^p}\right)\frac{\Pi\left(m^w\right)}{r}+\int_{-\infty}^{Q^{-1}\left(\frac{\Pi\left(m^w\right)/r}{\frac{c}{1-r}+m^p}\right)}Q(t)\,dt\\ &\le \left(1-\frac{c}{\frac{c}{1-r}+m^p}\right)\frac{\Pi\left(m^p\right)}{r}+\int_{-\infty}^{Q^{-1}\left(\frac{\Pi\left(m^p\right)/r}{\frac{c}{1-r}+m^p}\right)}Q(t)\,dt=W\left(m^p\right),\end{aligned}$$

where the first inequality follows from $\frac{dW}{dm}|_\Pi>0$ and the initial supposition $m^p>m^w$, while the second inequality from the definition of $m^p=\arg\max_m \Pi(a)$. So $W(m^w)<W(m^p)$, which contradicts the

definition of m^w. Thus, we must have $m^p \leq m^w$ when $c > c^h$. ■

Proof. (**Proposition 3**). From the main text, for each given m the profit-maximizing transaction fee $\tau^p(m)$ is defined implicitly by

$$\tau^p = \frac{Q(v(m) - m - c - \tau^p)}{Q'(v(m) - m - c - \tau^p)}.$$

Given that Q is log-concave, τ^p has less-than unity pass-through rate for any unit increase in $v - m$ (i.e. $\frac{d\tau^p}{d(v-m)} \in [0, 1]$). Thus, $v(m) - m - \tau^p(m)$ is increasing in $v(m) - m$ and it is maximized at m^p. To show $m^p \leq m^{sb}$, suppose by contradiction the reverse is true so $m^p > m^{sb}$. Then

$$\begin{aligned}
W\left(m^{sb}\right) &\leq \left(\tau^p\left(m^{sb}\right) + m^{sb}\right) Q\left(v\left(m^p\right) - m^p - \tau^p\left(m^p\right) - c\right) + \int_{-\infty}^{v(m^p) - m^p - \tau^p(m^p) - c} Q(t)\, dt \\
&< \left(\tau^p\left(m^p\right) + m^p\right) Q\left(v\left(m^p\right) - m^p - \tau^p\left(m^p\right) - c\right) + \int_{-\infty}^{v(m^p) - m^p - \tau^p(m^p) - c} Q(t)\, dt \\
&= W\left(m^p\right),
\end{aligned}$$

where the first inequality comes from the fact that $v(m) - m - \tau^p(m)$ is maximized at m^p; while the second inequality comes from τ^p being maximized at m^p and that $m^p > m^{sb}$. So, $W\left(m^{sb}\right) < W\left(m^p\right)$, which contradicts the definition of m^{sb}. ■

Proof. (**Proposition 4**). We first consider the case $c \geq c^h$, where $c^h > 0$ is given by 16. To show $m^p \leq m^{sb}$, suppose by contradiction $m^p > m^{sb}$. We proceed in three steps:

- (Step 1) $m^p > m^{sb}$ implies $v(m^p) - m^p < v(m^{sb}) - m^{sb}$;
- (Step 2) $m^p > m^{sb}$ and $v(m^p) - m^p < v(m^{sb}) - m^{sb}$ implies $r^p(m^{sb}) \geq r^p(m^p)$;
- (Step 3) $r^p(m^{sb}) \geq r^p(m^p)$ and $c \geq c^h$ implies $\tilde{W}(m^{sb}) < \tilde{W}(m^p)$, contradicting the definition of m^{sb}.

Step 1. Given $c > 0$, we can reframe the platform choice of proportional fee r as choosing the induced final product price p, through the one-to-one correspondence $p = m + \frac{c}{1-r}$. Then, for each given m the platform solves

$$\max_p \left(p - \frac{pc}{p - m}\right) Q(v(m) - p),$$

and leads to the optimal price $p^p(m)$ implicitly pinned down by

$$p^p = \frac{Q(v(m) - p^p)}{Q'(v(m) - p^p)} \left[1 + \frac{c}{p^p - m - c}\right]. \tag{17}$$

If $v(m^p) < v\left(m^{sb}\right)$ then the initial supposition of $m^p > m^{sb}$ immediately implies $v(m^p) - m^p < v\left(m^{sb}\right) - m^{sb}$, and we are done. If instead $v(m^p) > v\left(m^{sb}\right)$, then implicit function theorem on (17) shows $\frac{\partial p}{\partial v}$ and $\frac{\partial p}{\partial m}$ are both positive and so $p^p(m^p) > p^p(m^{sb})$. From the welfare function

$$\tilde{W}(m) = (p^p(m) - c)\, Q(v(m) - p^p(m)) + \int_{-\infty}^{v(m) - p^p(m)} Q(t)\, dt$$

we must have

$$v(m^p) - p^p(m^p) < v\left(m^{sb}\right) - p^p(m^{sb}) \tag{18}$$

as otherwise m^{sb} is not welfare-maximizing. The latter then implies $v(m^p) - m^p < v\left(m^{sb}\right) - m^{sb}$. To see this, suppose by contradiction $v(m^p) - m^p \geq v(m^{sb}) - m^{sb}$. Denote $t = v - p$ and rewrite (17) as

$$t = v - \frac{Q(t)}{Q'(t)}\left[1 + \frac{c}{v - m - t - c}\right],$$

where $v(m^p) - m^p \geq v(m^{sb}) - m^{sb}$ and $v(m^p) > v(m^{sb})$ would together imply $v(m^p) - p^p(m^p) > v(m^{sb}) - p^p(m^{sb})$, a contradiction to (18).

Step 2. Claim: *For any $m_2 \geq m_1$ such that $v(m_2) - m_2 \leq v(m_1) - m_1$, we have $r^p(m_1) \geq r^p(m_2)$ where $r^p(.)$ is given by (9).* We utilize the standard monotone comparative static result of Milgrom and Shannon (1994) by first introducing the following definition:

Definition 1 *(Single-crossing function). Let $\delta : \mathbb{R} \to \mathbb{R}$. Then $\delta(r)$ is single-crossing from above if for any $r_2 \geq r_1$ we have $\delta(r_1) \leq (<) 0 \implies \delta(r_2) \leq (<) 0$.*

By the standard monotone comparative static result of Milgrom and Shannon (1994), it suffices to show that the family of profit functions $\{\Pi(m,r)\}_{m \in \{m_1, m_2\}}$ obeys single-crossing difference from above, that is, $\frac{d\Pi}{dr}(m)$ is single-crossing from above. This is the same as showing the same property for

$$\delta(m) = \frac{Q\left(v(m) - m - \frac{c}{1-r}\right)}{Q'\left(v(m) - m - \frac{c}{1-r}\right)}\left(\frac{(1-r)^2}{c} + \frac{r}{m + \frac{c}{1-r}}\right) - r.$$

From the initial supposition of the claim and given strict log-concavity of Q, we have $\delta(m_2) \leq \delta(m_1)$, so that $\delta(m_2) \leq 0$ whenever $\delta(m_1) \leq 0$, so that $\frac{d\Pi}{dr}(m)$ is single-crossing from above with respect to $m \in \{m_1, m_2\}$. Given this claim, we let $m_1 = m^{sb}$ and $m_2 = m^p$, and we yield $r^p(m^{sb}) \geq r^p(m^p)$ as required.

Step 3. Similar to the proof of Proposition 2, we substitute for platform profit function $\tilde{\Pi}(m)$ in (10) to rewrite welfare function as

$$\tilde{W}(m) = \left(1 - \frac{c}{\frac{c}{1-r^p(m)} + m}\right)\frac{\tilde{\Pi}(m)}{r^p(m)} + \int_{-\infty}^{Q^{-1}\left(\frac{\tilde{\Pi}(m)/r^p(m)}{\frac{c}{1-r^p(m)}+m}\right)} Q(t)\,dt.$$

Recall that c^h is defined independently of the fee set by the platform, and it is defined such that for all $c > c^h$ and for all $m \in M(\Theta)$, $\tilde{W}(m)$ is increasing in m if $\tilde{\Pi}$ and r^p are held constant. Therefore,

$$\begin{aligned}
\tilde{W}(m^{sb}) &< \left(1 - \frac{c}{\frac{c}{1-r^p(m^{sb})} + m^p}\right)\frac{\tilde{\Pi}(m^{sb})}{r^p(m^{sb})} + \int_{-\infty}^{Q^{-1}\left(\frac{\Pi(m^w)/r^p(m^{sb})}{\frac{c}{1-r^p(m^{sb})}+m^p}\right)} Q(t)\,dt \\
&\leq \left(1 - \frac{c}{\frac{c}{1-r^p(m^{sb})} + m^p}\right)\frac{\tilde{\Pi}(m^p)}{r^p(m^{sb})} + \int_{-\infty}^{Q^{-1}\left(\frac{\Pi(m^p)/r^p(m^{sb})}{\frac{c}{1-r^p(m^{sb})}+m^p}\right)} Q(t)\,dt \\
&\leq \left(1 - \frac{c}{\frac{c}{1-r^p(m^p)} + m^p}\right)\frac{\tilde{\Pi}(m^p)}{r^p(m^p)} + \int_{-\infty}^{Q^{-1}\left(\frac{\Pi(m^p)/r^p(m^p)}{\frac{c}{1-r^p(m^p)}+m^p}\right)} Q(t)\,dt = \tilde{W}(m^p),
\end{aligned}$$

where the first inequality follows from the construction of c^h and the initial supposition $m^p > m^w$, while the second inequality follows from the definition of $m^p = \arg\max_m \Pi(a)$, and the third inequality follows from $r^p(m^{sb}) \geq r^p(m^p)$ and the observation that $\tilde{W}$ is decreasing in r. So $\tilde{W}(m^{sb}) < \tilde{W}(m^p)$, which contradicts the definition of m^w. Thus, we must have $m^p \leq m^w$ when $c \geq c^h$.

Consider the case of $c \to 0$. From (9) we note that $c \to 0$ implies $r^p \to 1$, so that $r^p(m)$ is independent of m hence it is also independent of a. Therefore, r^p is the same for the profit-maximizing platform and the social planner, and it follows from Proposition 2 that $m^p \geq m^w = m^{sb}$ as required. ■

Proof. (Corollary 1). Given that the platform fully extracts seller surplus, we use $\Pi(m) = mQ(v(m) - m - c)$ so we rewrite the welfare function as:

$$W(m) = \Pi(m) + \int_{-\infty}^{Q^{-1}\left(\frac{\Pi(m)}{m}\right)} Q(t)\,dt.$$

By contradiction, suppose $m^p < m^{sb}$, then

$$\begin{aligned} W\left(m^{sb}\right) &\leq \Pi(m^p) + \int_{-\infty}^{Q^{-1}\left(\frac{\Pi(m^p)}{m^{sb}}\right)} Q(t)\,dt \\ &< \Pi(m^p) + \int_{-\infty}^{Q^{-1}\left(\frac{\Pi(m^p)}{m^p}\right)} Q(t)\,dt = W(m^p), \end{aligned}$$

which is a contradiction to the definition of m^{sb}. ■

Proof. (Proposition 5). We focus on a two-part tariff in the form of a seller participation fee and a per-transaction fee τ because the case of a proportional fee r can be proven similarly. Recall that for each given m, the profit-maximizing platform chooses

$$\tau^p(m) = \max\{0, p^*(m) - m - c\},$$

where p^* is defined by

$$p^* - c = \frac{Q(v(m) - p^*)}{Q'(v(m) - p^*)},$$

It is useful to note that p^* exhibits incomplete pass-through so that $\frac{dp^*}{dv} \in (0, 1]$, which implies that $v - p^*$ is increasing in v and so $v - p^*$ is maximized when $m^* \equiv \arg\max_m v(m^*)$.

To show $m^p \geq m^{sb}$, suppose by contradiction the reverse is true so that $m^p < m^{sb}$. In what follows we show that this leads to a contradiction. We consider four cases according to the value of $\tau^p(m)$. (i) If $\tau^p(m^p) = \tau^p\left(m^{sb}\right) = 0$, then the analysis is equivalent to the case of pure participation fee so $m^p < m^{sb}$ contradicts Corollary 1. (ii) Suppose $\tau^p(m^p) > \tau^p\left(m^{sb}\right) = 0$, so that $m^p = m^*$. Then,

$$\begin{aligned} \tilde{W}\left(m^{sb}\right) &= m^{sb}Q\left(v(m^{sb}) - m^{sb} - c\right) + \int_{-\infty}^{v(m^{sb})-m^{sb}-c} Q(t)\,dt \\ &\leq m^{sb}Q\left(v(m^{sb}) - p^*(m^{sb})\right) + \int_{-\infty}^{v(m^{sb})-p^*(m^{sb})} Q(t)\,dt \\ &\leq m^{sb}Q(v(m^p) - p^*(m^p)) + \int_{-\infty}^{v(m^p)-p^*(m^p)} Q(t)\,dt \\ &< m^pQ(v(m^p) - p^*(m^p)) + \int_{-\infty}^{v(m^p)-p^*(m^p)} Q(t)\,dt = \tilde{W}(m^p). \end{aligned}$$

Here, the first inequality is due to $p^*\left(m^{sb}\right) - c \leq m^{sb}$ as implied by $\tau^p\left(m^{sb}\right) = 0$. The second inequality comes from $v(m) - p^*(m)$ being maximized at $m^p = m^*$; while the last inequality comes from $m^p < m^{sb}$. So, $\tilde{W}\left(m^{sb}\right) < \tilde{W}(m^p)$, which contradicts the definition of m^{sb} being the social planner's optimal choice.

(iii) If $\tau^p(m^s) > \tau^p(m^p) = 0$,

$$\begin{aligned}
\tilde{W}\left(m^{sb}\right) &= \left(\tau^p(m^{sb}) + m^{sb}\right) Q\left(v(m^{sb}) - m^s - \tau^p(m^{sb}) - c\right) + \int_{-\infty}^{v(m^{sb})-m^{sb}-\tau^p(m^{sb})-c} Q(t)\,dt \\
&< m^{sb} Q\left(v(m^{sb}) - m^{sb} - c\right) + \int_{-\infty}^{v(m^{sb})-m^{sb}-c} Q(t)\,dt \\
&\leq \tilde{W}(m^p),
\end{aligned}$$

where the first inequality comes from $\tau^p(m^p) = 0$ and welfare being strictly decreasing in τ, while the second inequality is established as in Corollary 1. So $\tilde{W}\left(m^{sb}\right) < \tilde{W}(m^p)$, which contradicts the definition of m^{sb}. (iv) Finally, if $\tau^p(m^p) > 0$ and $\tau^p(m^{sb}) > 0$, so that $p^*(m^p) - c > m^p$ and $p^*(m^{sb}) - c > m^{sb}$. Then,

$$\begin{aligned}
\tilde{W}\left(m^{sb}\right) &= \left(p^*(m^{sb}) - c\right) Q\left(v(m^{sb}) - p^*(m^{sb}) - c\right) + \int_{-\infty}^{v(m^{sb})-p^*(m^{sb})} Q(t)\,dt \\
&< \left(p^*(m^{sb}) - c\right) Q\left(v(m^p) - p^*(m^p) - c\right) + \int_{-\infty}^{v(m^p)-p^*(m^p)} Q(t)\,dt \\
&\leq \left(p^*(m^p) - c\right) Q\left(v(m^p) - p^*(m^p) - c\right) + \int_{-\infty}^{v(m^p)-p^*(m^p)} Q(t)\,dt = \tilde{W}(m^p),
\end{aligned}$$

where the first inequality comes from $v(m) - p^*(m)$ being maximized at $m^p = m^*$ (recall m^* is unique so $m^{sb} > m^p$ implies $m^{sb} \neq m^*$); while the second inequality comes from p^* being maximized at $m^p = m^*$. So $\tilde{W}\left(m^{sb}\right) < \tilde{W}(m^p)$, which contradicts the definition of m^{sb}. ■

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Online appendix: Platform governance

Tat-How Teh*

This online appendix contains omitted details from the main paper and the analysis for the model extensions discussed in Section 6.

A Derivations of Examples 1-3

This section provides the derivations for the micro-founded examples in Section 2. In what follows we do not specify the exact fee instrument used by the platform. Instead, we focus on deriving how the platform's governance design influences the buyer-seller interactions in each of the applications.

A.1 Entry regulation and variety choice by platform

The model in Example 1 can be summarized by the following sequence of events: (i) The platform announces the number of admitted sellers a; (ii) Upon observing a, buyers make the decision to join the platform; (iii) The admitted sellers set their prices, then buyers observe the prices and match values, and purchase accordingly. We focus on symmetric pure strategy Nash equilibrium where all admitted sellers set the same price p (for any a chosen by the platform).

Let Q be the number of participating buyers, which is exogenous from sellers' point of view. To derive seller pricing, consider a deviating seller i who sets price $p_i \neq p$, its product is purchased if $\epsilon_i - p_i \geq \max_{j\neq i, j\leq a}\{\epsilon_j - p\}$ implying that the demand for seller i's product is

$$Q_i(p_i) = Q \times \Pr\left(\epsilon_i - p_i \geq \max_{j\neq i, j<a}\{\epsilon_j - p\}\right) = Q \times \int_0^\infty (1 - F(\epsilon - p + p_i))\, dF(\epsilon)^{a-1},$$

and profit function $(p-c)\,Q_i(p_i)$. Log-concavity of f implies log-concavity of $Q_i(p_i)$, so that the symmetric equilibrium price in this environment can be derived from the usual first-order condition, i.e.

$$p = c - \frac{Q_i(p_i)}{dQ_i(p_i)/dp_i}|_{p_i=p} = c + \frac{1}{a\int_0^\infty f(\epsilon)\, dF^{a-1}(\epsilon)}.$$

As stated in the main text, a buyer will join the platform if $\mathbf{E}(\max_{i=1,\ldots,a}\{\epsilon_i\}) - p > d$, so that $Q = G(\mathbf{E}(\max_{i=1,\ldots,a}\{\epsilon_i\}) - p)$.

A.2 Quality control by platform

The model in Example 2 can be summarized by the following sequence of events: (i) The platform announces its quality standard a; (ii) Sellers with $q_i \geq a$ join the platform and set their prices; (iii) Without observing seller prices, buyers decide whether to search given the respective realized buyer-specific match component x. Buyers that initiate search carry out sequential search. We focus on symmetric Perfect Bayesian Equilibria (PBE) where all sellers set the same price p. As is standard in the search literature, buyers keep the same (passive) beliefs about the distribution of future prices on and off the equilibrium path.

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We first derive buyers' search strategy for a given a set by the platform. Define the reservation value $V(a)$ as the solution to

$$\mathbf{E}(q_i|q_i \geq a) \int_V^{\bar{\epsilon}} (\epsilon - V) \, dF(\epsilon) = s. \tag{A.1}$$

The left-hand side of (A.1) represents the incremental expected benefit from one more search, while the right-hand side represents the incremental search cost. There is, at most, one solution to (A.1) since the left-hand side is strictly decreasing in v.

It is well known from Weitzman (1979) that buyers' optimal search rule in this environment is stationary and described by the standard cutoff rule. When searching, each buyer employs the following strategy: (i) she stops and buys form seller i if the product is non-defective and $x+\epsilon_i-p_i \geq x+V(a)-p$; (ii) she continues to search the next seller otherwise. Following the standard result, the buyer's expected surplus from initiating a search is $x + V(a) - p$. Then, buyers with $x < p - V(a)$ expect no surplus gain from costly search and thus will not join the platform, while buyers with $x \geq p - V(a)$ will do so. Provided that search cost is sufficiently small, there is a symmetric price equilibrium where a strictly positive measure of buyers join the platform.

Compared to a standard search model, notice from (3) that a search pool with a higher expected quality $\mathbf{E}(q_i|q_i \geq a)$ is analogous to a lower "effective search cost" for buyers. This reflects that each buyer searches less and consequently incurs a lower total expected search cost of $s/\mathbf{E}(q_i|q_i \geq a)$ before reaching a non-defective match. Given that $\mathbf{E}(q_i|q_i > a)$ increases with a, a higher quality standard set by the platform is analogous to reducing the effective search cost of buyers. Thus, it follows that $V(a)$ is an increasing function of a.

From the buyer search rule above, derivation of demand is straightforward. The mass of buyers initiating search is

$$Q = 1 - G(p - V(a)),$$

which is exogenous from each firm's point of view. Conditional on these buyers, the demand of a deviating firm i with type q_i follows the standard search model and it is given by

$$q_i (1 - F(V(a) - p + p_i)) \sum_{z=0}^{\infty} F(V(a))^z = \left(\frac{1 - F(V(a) - p + p_i)}{1 - F(V(a))} \right) q_i.$$

The log-concavity assumption on $1 - F$ ensures that the usual first-order condition determines a unique optimal price. The symmetric equilibrium price is given by

$$p = c + \frac{1 - F(V(a))}{f(V(a))}.$$

Note $1 - F$ being log-concavity implies that $M(a)$ is decreasing in a. This reflects that a higher quality standard reduces the effective search cost of buyers, which leads to a more elastic product demand.

A.3 On-platform search cost

The model in Example 3 can be summarized by the following sequence of events: (i) The platform sets and announces its search design a (recall that on-platform search cost s is decreasing in a); (ii) Sellers set their prices; (iii) Without observing seller prices and any product match values, buyers decide whether to join the platform given their respective realized joining cost d; (iv) Buyers that join the platform observe their prior match values and prices of each product, and search sequentially. We focus on symmetric pure-strategy Nash equilibrium where all sellers set the same price p. The exposition below follow closely that by Choi et. al. (2018), and we refer readers to their paper for further details.

To characterize a buyer's optimal search strategy, we can again utilize Weitzman (1979)'s solution by first defining the reservation value z^* as

$$\int_{z^*}^{\bar{z}} (z_i - z^*) \, dH(z_i) = s(a).$$

After realizing prior match values $(\epsilon_1, ..., \epsilon_n)$ and observing prices $(p_1, ..., p_n)$, a buyer's search strategy can be characterized as follows: (i) she inspects sellers in the descending order of $\epsilon_i + z^* - p_i$; (ii) let N be the set of sellers the buyer has visited so far, then she stops and takes the best available option by the point if $\max_{i \in N} \{\epsilon_i + z_i - p_i\} > \max_{j \notin N} \{\epsilon_j + z^* - p_j\}$; (iii) she continues searching otherwise. From Weitzman's solution, Theorem 1 in Choi et al. (2018) provides a way to characterize each buyer's eventual purchase decision:

Remark 1 *(Choi et al., 2018) For each i, define $w_i \equiv \epsilon_i + \min\{z_i, z^*\}$. Given $(\epsilon_1, ..., \epsilon_n)$, $(z_1, ..., z_n)$, and $(p_1, ..., p_n)$, a buyer eventually purchases product i if and only if $w_i - p_i > w_j - p_j$ for all $j \neq i$.*

As a direct corollary of Remark 1, it can be shown that the expected surplus from participating in the market is $\mathbf{E}(\max_{i=1,...,n} \{w_i - p_i\})$. A buyer joins the platform if her joining cost is lower than the expected surplus. This implies that in the symmetric price equilibrium to be characterized below, the mass of buyers joining the platform is $Q = G(\mathbf{E}(\max_{i=1,...,n}\{w_i\}) - p)$.

Let $\bar{H}$ denote the cdf of the new random variable $w_i \equiv \epsilon_i + \min\{z_i, z^*\}$ that is common for all i, and let $\bar{h}$ denotes the corresponding density function. Consider a deviating seller i who sets $p_i \neq p$. Owing to Remark 1, we can write down seller i's demand function as

$$\Pr\left(w_i - p_i \geq \max_{j \neq i} \{w_j - p\}\right) = \int_{-\infty}^{\infty} \left(1 - \bar{H}(w - p + p_i)\right) d\bar{H}^{n-1}(w).$$

It can be shown that the demand function is log-concave in p_i provided that the variance of ϵ_i is large enough. Consequently, the symmetric equilibrium price in this environment can be derived from the usual first-order condition as $p = c + M(a)$, where

$$M(a) \equiv \frac{1/n}{\int_{-\infty}^{\infty} \bar{h}(w) \, d\bar{H}^{n-1}(w)}.$$

Note that the distribution function $\bar{H}(w_i)$ is a function of a because the definition of variable w_i depends on z^*, which in turns depends on $s(a)$. Choi et al. show that the distribution of w_i becomes more "dispersed" when $s(a)$ increases, which allows them to show that $M(a)$ is decreasing in search cost, meaning that a higher search cost intensifies seller competition.

B Alternative welfare benchmark

In this section, we replicate the analysis in Section 4 of the main text by considering the alternative welfare benchmark where the social planner controls both the fee level charged by the platform as well as the governance design.

☐ **Per-transaction fee**. The social planner sets per-transaction fee at $\tau = 0$ to minimize deadweight losses, so that the welfare function in (8) becomes

$$\tilde{W}(m) = mQ(v(m) - m - c) + \int_{-\infty}^{v(m)-m-c} Q(t) \, dt. \tag{B.1}$$

Then:

Proposition B.1 *If a social planner can control the platform's governance design and the per-transaction fee τ, then the planner prefers a design that is associated with less on-platform competition than the platform (i.e. $m^p \leq m^{sb}$).*

Proof. By contradiction, suppose instead $m^p > m^{sb}$. We have

$$\begin{aligned}\tilde{W}\left(m^{sb}\right) &\leq m^{sb} Q\left(v\left(m^p\right) - m^p - c\right) + \int_{-\infty}^{v(m^p)-m^p-c} Q\left(t\right) dt \\ &< m^p Q\left(v\left(m^p\right) - m^p - c\right) + \int_{-\infty}^{v(m^p)-m^p-c} Q\left(t\right) dt \\ &= \tilde{W}\left(m^p\right),\end{aligned}$$

where the first inequality follows from the definition of m^p in (4), while the second inequality follows from the initial supposition that $m^p > m^w$. Obviously, $W\left(m^w\right) < W\left(m^p\right)$ contradicts the definition of m^{sb}, and so we must have $m^p \leq m^{sb}$. ■

□ **Proportional fee**. The social planner sets percentage fee at $r = 0$ to minimize deadweight losses, so that the welfare function in (11) becomes (B.1). Then:

Proposition B.2 *If a social planner can control the platform's governance design and the proportional fee r, then:*

1. *If $c \to 0$ then the planner prefers a design that is associated with more on-platform competition than the platform (i.e. $m^p \geq m^{sb}$).*
2. *If $c \geq c^h$ then the reverse is true (i.e. $m^p \leq m^{sb}$).*

Proof. In this proof, it is useful to label $m^p\left(r\right)$ as the profit-maximizing design for each given r, and the profit-maximizing r as $r^p \geq 0$. We first prove the following claim:

Claim: *$m^p\left(r\right)$ is decreasing in r and becomes independent of r if $c \to 0$.* Write

$$\Pi\left(m, r\right) = r\left(m + \frac{c}{1-r}\right) Q\left(v - m - \frac{c}{1-r}\right)$$

We want to show $\arg\max_m \Pi\left(m, r\right)$ is decreasing in r. By Milgrom and Shannon (1994), showing $\arg\max_m \Pi\left(m, r\right)$ is decreasing in r is equivalent to showing that the family of profit functions $\left\{\Pi\left(m, r\right)\right\}_{r\in[0,1]}$ obeys single-crossing difference from above, that is: for all $m_1 \leq m_2$,

$$\delta\left(r\right) \equiv \Pi\left(m_2, r\right) - \Pi\left(m_1, r\right)$$

is single-crossing from above with respect to r (See the proof of Proposition 4 for the definition of single-crossing functions). One can verify that the sign of $\delta\left(r\right)$ is the same as the sign of the following expression:

$$\left(m_2 - m_1\right) + \left(m_1 + \frac{c}{1-r}\right)\left[1 - \frac{Q\left(v\left(m_1\right) - m_1 - \frac{c}{1-r}\right)}{Q\left(v\left(m_2\right) - m_2 - \frac{c}{1-r}\right)}\right]. \tag{B.2}$$

If $v\left(m_1\right) - m_1 \leq v\left(m_2\right) - m_2$ then $\delta\left(r\right) \geq 0$ for all r, so that single-crossing condition holds trivially. Suppose instead $v\left(m_1\right) - m_1 \geq v\left(m_2\right) - m_2$, then the second component in (B.2) is negative and it is decreasing in r because log-concavity of Q implies that $\frac{Q\left(v(m_1)-m_1-\frac{c}{1-r}\right)}{Q\left(v(m_2)-m_2-\frac{c}{1-r}\right)}$ is increasing in r whenever $v\left(m_1\right) - m_1 \geq v\left(m_2\right) - m_2$. Hence, (B.2) is decreasing in r in this case, implying that the single-crossing condition holds. Combining both cases, we conclude that $\delta\left(r\right)$ is indeed single-crossing so that

$m^p(r) = \arg\max_m \Pi(m, r)$ is decreasing in r. Moreover, we note from (B.2) that if $c \to 0$, then $\delta(r)$ becomes independent of r.

We are now ready to prove the proposition. Consider first the case of $c \geq c^h$, where c^h is defined by (16). From Proposition 2, if we let $r = 0$ in the proposition statement, then we know $m^{sb} \geq m^p(0)$. Therefore, it remains to show $m^p(0) \geq m^p(r^p)$, which follows directly from the claim above that $m^p(r)$ is decreasing in r and $r^p \geq 0$. When $c \to 0$, $m^p(r)$ becomes independent of r so $\lim_{c\to 0} m^p(r^p) = m^p(0)$, while Proposition 2 implies $m^p(0) \geq m^{sb}$ when $c \leq c^l$ (where $c^l > 0$ is defined by (15)) as required. ■

□ **Participation fee and two-part tariff**. Given that a participation fee is a welfare-neutral lump-sum transfer, Corollary 1 remains valid under this alternative welfare benchmark. Consider instead two-part tariffs. The social planner again sets the transaction fee component (either a per-transaction fee or a percentage fee) at zero so that the welfare function is (B.1). Then:

Proposition B.3 *If a social planner can control the platform's governance design and the two-part tariff charged by the platform, then the planner prefers a design that is associated with more on-platform competition than the platform (i.e. $m^p \geq m^{sb}$).*

Proof. Denote $m^* \equiv \arg\max v(m)$ and $p^*(m)$ implicitly defined by $p^* - c = \frac{Q(v(m)-p^*(m))}{Q'(v(m)-p^*(m))}$. Consider first the case that $m^* \leq p^*(m^*) - c$. Then the profit-maximizing platform chooses $m^p = m^*$, and τ that satisfies first-order condition

$$\tau + m^* = \frac{Q(v(m^*) - c - \tau - m^*)}{Q'(v(m^*) - c - \tau - m^*)} \text{ under per-transaction fee}$$

or r that satisfies

$$\frac{c}{1-r} + m^* - c = \frac{Q\left(v(m^*) - \frac{c}{1-r} - m^*\right)}{Q'\left(v(m^*) - \frac{c}{1-r} - m^*\right)} \text{ under proportional fee.}$$

These fees induce the monopoly price level $p^*(m^*)$, and the fees must be non-negative given the supposition $m^* \leq p^*(m^*) - c$. To show $m^p \geq m^{sb}$, suppose instead the reverse is true so that $m^p < m^{sb}$. Given that $\tau^{sb} = r^{sb} = 0$, we have

$$\begin{aligned}
\tilde{W}\left(m^{sb}\right) &\leq m^{sb} Q\left(v(m^p) - m^{sb} - c\right) + \int_{-\infty}^{v(m^p)-m^{sb}-c} Q(t)\,dt \\
&< m^p Q(v(m^p) - m^p - c) + \int_{-\infty}^{v(m^p)-m^p-c} Q(t)\,dt \\
&= \tilde{W}(m^p),
\end{aligned}$$

where the first inequality follows from the definition of $m^p = m^*$, while the second inequality follows from the initial supposition that $m^p < m^{sb}$ and that $d\tilde{W}/dm \leq 0$. Therefore, $\tilde{W}\left(m^{sb}\right) \leq \tilde{W}(m^p)$, which contradicts the definition of m^{sb}. Hence, we must have $m^p \geq m^{sb}$. In the case where $m^* > p^*(m^*) - c$, $p^*(m^*)$ cannot be implemented with $\tau \geq 0$ or $r \geq 0$. The log-concavity assumption on Q implies that the platform does best by setting $\tau = 0$ or $r = 0$, so that the platform profit is equivalent to the seller surplus, and the result follows from Corollary 1. ■

C Extension: Incomplete pass-through and market coverage

In this section, we extend the framework in Section 2 by generalizing the markup function $M = M(p, a)$ and the aggregate demand function Q.

Following the description in Section 6, we write the symmetric equilibrium price equation in (1) as

$$p = c + M(p, a). \tag{C.1}$$

The markup function $M(p, a)$ is continuous with respect to a for all $a \in \Theta$, while it is decreasing and differentiable with respect to p. The latter assumption ensures that a unique p is defined by (C.1) for each given c and a, so that we can write $p = p(a)$.

Throughout, instead of working directly with function $M(p, a)$, it is simpler to work with the "composite markup function" defined by

$$\bar{M}(a) \equiv M(p(a), a),$$

where $p(a)$ is given by (C.1). We have the following properties $\bar{M}(a)$:

- $\bar{M}(a)$ is decreasing in c:
$$\frac{d\bar{M}}{dc} = \frac{dM}{dp}\frac{dp}{dc} = \frac{\frac{dM}{dp}}{1 - \frac{dM}{dp}} \in [-1, 0].$$
- $\bar{M}(a)$ is increasing in a if M is increasing in a, and conversely $\bar{M}$ is decreasing in a if M is decreasing in a. To see this:
$$\frac{d\bar{M}}{da} = \frac{dM}{dp}\frac{dp}{da} + \frac{dM}{da} = \frac{1}{1 - \frac{dM}{dp}}\frac{dM}{da}.$$

As stated in Section 6, we consider two possibilities for the aggregate demand function, each with the following associated assumptions.

- **Case 1,** $Q = Q(V(a) - p)$: As in the baseline model, $Q(.)$ is strictly increasing, continuously differentiable, and log-concave.
- **Case 2,** $Q = Q(V(a), p)$: As stated in the main text, $Q(., .)$ is strictly increasing in its first argument, strictly decreasing in its second argument, log-concave and twice continuously differentiable in both arguments.

C.1 Exogenous per-transaction fee

For each given per-transaction fee τ and governance design a, we write the symmetric equilibrium price equation in (C.1) as $p = p(a)$ that implicitly solves

$$p = c + \tau + M(p, a).$$

Denote $\bar{M}(a) \equiv M(p(a), a)$. We prove the following two propositions that are analogous to Proposition 2.

Proposition C.1 *Suppose $Q = Q(V(a) - p)$ and the platform charges an exogenous per-transaction fee τ. The profit-maximizing governance design induces excessive on-platform competition (i.e. $\bar{M}(a^p) \leq \bar{M}(a^w)$).*

Proof. Given the aggregate demand function, the platform's profit is

$$\Pi(a) = \tau Q\left(V(a) - \bar{M}(a) - \tau - c\right).$$

We have $a^p \equiv \arg\max_a \Pi = \arg\max_a \left\{V(a) - \bar{M}(a)\right\}$. The welfare function is

$$W(a) = \left(\tau + \bar{M}(a)\right) Q\left(V(a) - \bar{M}(a) - c - \tau\right) + \int_{-\infty}^{V(a)-\bar{M}(a)-c-\tau} Q(t)\, dt.$$

Let $a^w \equiv \arg\max_a W(a)$. The result follows from the proof of Proposition 1. ■

Proposition C.2 *Suppose $Q = Q(V(a), p)$ and the platform charges an exogenous per-transaction fee τ. The profit-maximizing governance design induces excessive on-platform competition (i.e. $\bar{M}(a^p) \leq \bar{M}(a^w)$).*

Proof. Given the aggregate demand function, the platform's profit is

$$\Pi(a) = \tau Q\left(V(a), \bar{M}(a) + \tau + c\right).$$

We have $a^p \equiv \arg\max_a \Pi = \arg\max_a Q\left(V(a), \bar{M}(a) + \tau + c\right)$.The welfare function is

$$W(a) = \left(\tau + \bar{M}(a)\right) Q\left(V(a), \bar{M}(a) + \tau + c\right) + \int_{\bar{M}(a)+\tau+c}^{\infty} Q(V(a), t)\, dt.$$

Let $a^w \equiv \arg\max_a W(a)$. Suppose by contradiction $\bar{M}(a^p) > \bar{M}(a^w)$, which implies $V(a^p) > V(a^w)$ by the definition of a^p. The welfare function is

$$\begin{aligned} W(a^w) &= \left(\tau + \bar{M}(a^w)\right) Q\left(V(a^w), \bar{M}(a^w) + c + \tau\right) + \int_{\bar{M}(a^w)+\tau+c}^{\infty} Q(V(a^w), t)\, dt \\ &< \left(\tau + \bar{M}(a^p)\right) Q\left(V(a^w), \bar{M}(a^w) + c + \tau\right) + \int_{\bar{M}(a^p)+\tau+c}^{\infty} Q(V(a^w), t)\, dt \\ &< \left(\tau + \bar{M}(a^p)\right) Q\left(V(a^p), \bar{M}(a^p) + c + \tau\right) + \int_{\bar{M}(a^p)+\tau+c}^{\infty} Q(V(a^p), t)\, dt \\ &= W(a^p), \end{aligned}$$

where the first inequality follows from the following inequality that utilizes $Q(.,.)$ being strictly decreasing in its second argument:

$$\begin{aligned} & \left(\bar{M}(a^p) - \bar{M}(a^w)\right) Q\left(V(a^w), \bar{M}(a^w) + c + \tau\right) - \int_{\bar{M}(a^w)+\tau+c}^{\bar{M}(a^p)+\tau+c} Q(V(a^w), t)\, dt \\ \geq\ & \left(\bar{M}(a^p) - \bar{M}(a^w)\right) Q\left(V(a^w), \bar{M}(a^w) + c + \tau\right) - \int_{\bar{M}(a^w)+\tau+c}^{\bar{M}(a^p)+\tau+c} Q\left(V(a^w), \bar{M}(a^w) + \tau + c\right) dt \\ =\ & 0, \end{aligned}$$

while the second inequality follows from $V(a^p) > V(a^w)$ and the definition of $a^p \equiv \arg\max Q$. Hence, we have $W(a^p) > W(a^w)$, contradicting the definition of a^w. ■

C.2 Exogenous proportional fee

For each given proportional fee r and governance design a, we write the symmetric equilibrium price equation in (C.1) as $p = p(a)$ that implicitly solves

$$p = \frac{c}{1-r} + M(p, a). \tag{C.2}$$

Again, we define $\bar{M}(a) \equiv M(p(a), a)$. We prove the following two propositions that are analogous to Proposition 2.

Proposition C.3 *(Exogenous proportional fee) Suppose $Q = Q(V(a) - p)$ and the platform charges an exogenous proportional fee r. There exist thresholds c^l and c^h, where $0 < c^l \leq c^h$, such that:*

1. *If $c < c^l$, the profit-maximizing governance design induces insufficient on-platform competition (i.e. $\bar{M}(a^p) \geq \bar{M}(a^w)$);*
2. *If $c > c^h$, the reverse is true (i.e. $\bar{M}(a^p) \leq \bar{M}(a^w)$).*

Proof. The platform's profit and the welfare function are

$$\begin{aligned} \Pi(a) &= rpQ(V(a) - p) \\ W(a) &= (p-c)Q(V(a)-p) + \int_{-\infty}^{V(a)-p} Q(t)\,dt, \end{aligned}$$

where $p = p(a)$ is given by (C.2). After substituting in $\Pi(a)$ into the the welfare function, the proof proceeds similarly to Proposition 2 up until the step where we establish how the (substituted) welfare function changes with markup (while holding Π constant). We have

$$\frac{dW}{d\bar{M}}|_\Pi = \frac{\Pi(a)/r}{\left(\frac{c}{1-r} + \bar{M}(a)\right)^2}\left[c - \frac{Q\left(V(a) - \frac{c}{1-r} - \bar{M}(a)\right)}{Q'\left(V(a) - \frac{c}{1-r} - \bar{M}(a)\right)}\right].$$

Define $a^h \equiv \arg\max_a \{V(a)\}$ and $a^l \equiv \arg\min_a \left\{V(a) - \bar{M}(a)|_{c=0}\right\}$. Then:

- Let c^l be the unique solution to $c - \frac{Q\left(v(a^l) - \bar{M}(a^l)|_{c=0} - \frac{c}{1-r}\right)}{Q'\left(v(a^l) - \bar{M}(a^l)|_{c=0} - \frac{c}{1-r}\right)} = 0$. The existence and uniqueness of c^l follows from the intermediate value theorem given log-concavity of Q (so that Q/Q' is increasing) and Q being continuously differentiable. By construction, for all $c < c^l$ and for all $a \in \Theta$, we have

$$\begin{aligned} c - \frac{Q\left(V(a) - \frac{c}{1-r} - \bar{M}(a)\right)}{Q'\left(V(a) - \frac{c}{1-r} - \bar{M}(a)\right)} &\leq c - \frac{Q\left(V(a) - \frac{c}{1-r} - \bar{M}(a)|_{c=0}\right)}{Q'\left(V(a) - \frac{c}{1-r} - \bar{M}(a)|_{c=0}\right)} \\ &\leq c - \frac{Q\left(V(a^l) - \frac{c}{1-r} - \bar{M}(a^l)|_{c=0}\right)}{Q'\left(V(a^l) - \frac{c}{1-r} - \bar{M}(a^l)|_{c=0}\right)} \\ &< c^l - \frac{Q\left(V(a^l) - \frac{c^l}{1-r} - \bar{M}(a^l)|_{c=0}\right)}{Q'\left(V(a^l) - \frac{c^l}{1-r} - \bar{M}(a^l)|_{c=0}\right)} = 0, \end{aligned}$$

  where the first inequality follows from $\bar{M}$ being decreasing in c, the second inequality follows from the definition of a^l, while the last inequality follows from $c < c^l$. Therefore, $\frac{dW}{dM}|_\Pi < 0$ for all $c < c^l$ and $a \in \Theta$.

- Let c^h be the unique solution to $c - \frac{Q\left(v\left(a^h\right)-\frac{c}{1-r}\right)}{Q'\left(v\left(a^h\right)-\frac{c}{1-r}\right)} = 0$. The existence and uniqueness of c^h follows from the same reasons as before. By construction, for all $c > c^h$ and $a \in \Theta$, we have

$$\begin{aligned} c - \frac{Q\left(V\left(a\right) - \frac{c}{1-r} - \bar{M}\left(a\right)\right)}{Q'\left(V\left(a\right) - \frac{c}{1-r} - \bar{M}\left(a\right)\right)} &\geq c - \frac{Q\left(V\left(a\right) - \frac{c}{1-r}\right)}{Q'\left(V\left(a\right) - \frac{c}{1-r}\right)} \\ &\geq c - \frac{Q\left(v\left(a^h\right) - \frac{c}{1-r}\right)}{Q'\left(v\left(a^h\right) - \frac{c}{1-r}\right)} \\ &> c^h - \frac{Q\left(v\left(a^h\right) - \frac{c^h}{1-r}\right)}{Q'\left(v\left(a^h\right) - \frac{c^h}{1-r}\right)} = 0, \end{aligned}$$

where the first inequality follows from $\bar{M} \geq 0$, the second inequality follows from the definition of a^h, while the last inequality follows from $c > c^h$. Therefore, $\frac{dW}{dM}|_{\Pi} > 0$ for all $c > c^h$ and $a \in \Theta$.

Using the defined thresholds c^l and c^h, the remaining parts of the proof are the same as the proof of Proposition 2. ■

As mentioned in the main text, for the case where $Q = Q\left(V\left(a\right), p\right)$ it is useful to introduce the following assumption:

In order to apply the same monotone comparative static arguments that establish our results, we require the following assumption on the demand function:

Assumption 1 *For all V and p, the ratio $\Psi \equiv \frac{\partial Q(V,p)/\partial p}{\partial Q(V,p)/\partial V} \leq 0$ is (weakly) decreasing in p.*

Assumption 1 is obviously satisfied in the baseline analysis where $Q\left(V\left(a\right), p\right) = Q\left(V\left(a\right) - p\right)$, in which $\Psi = -1$. Another sufficient condition for the assumption is for $Q\left(V, p\right)$ to be sub-modular and concave in p, so that the numerator of Ψ increases with p while the denominator decreases with p. A micro-foundation where the assumption holds is entry regulation by platform in the Perloff-Salop model with a binding outside option, provided that the density function of product match values is sufficiently log-linear.

Proposition C.4 *(Exogenous proportional fee) Suppose $Q = Q\left(V\left(a\right), p\right)$ and $\frac{\partial Q(V,p)}{\partial V}$ is log-concave in p. If the platform charges an exogenous proportional fee r. There exist thresholds c^l and c^h, where $0 < c^l \leq c^h$, such that:*

1. *If $c < c^l$ and Assumption 1 holds, the profit-maximizing governance design induces insufficient on-platform competition (i.e. $\bar{M}\left(a^p\right) \geq \bar{M}\left(a^w\right)$);*
2. *If $c > c^h$, the reverse is true (i.e. $\bar{M}\left(a^p\right) \leq \bar{M}\left(a^w\right)$).*

Proof. The platform's profit and the welfare function are

$$\begin{aligned} \Pi\left(a\right) &= rpQ\left(V\left(a\right), p\right) \\ W\left(a\right) &= \left(p - c\right) Q\left(V\left(a\right), p\right) + \int_p^\infty Q\left(V\left(a\right), t\right) dt, \end{aligned}$$

where $p = p\left(a\right)$ is given by (C.2). From

$$\Pi\left(a\right) = r\left(\frac{c}{1-r} + \bar{M}\left(a\right)\right) Q\left(V\left(a\right), \frac{c}{1-r} + \bar{M}\left(a\right)\right),$$

we let Q_1^{-1} be the inverse function of Q with respect to the first argument to write

$$V(a) = Q_1^{-1}\left(\frac{\Pi(a)/r}{\frac{c}{1-r}+\bar{M}(a)}; \frac{c}{1-r}+\bar{M}(a)\right).$$

Then, rewrite the welfare function as

$$W(a) = \left(1-\frac{c}{\frac{c}{1-r}+\bar{M}(a)}\right)\frac{\Pi(a)}{r} + \int_{\frac{c}{1-r}+\bar{M}(a)}^{\infty} Q\left(Q_1^{-1}\left(\frac{\Pi(a)/r}{\frac{c}{1-r}+\bar{M}(a)}; \frac{c}{1-r}+\bar{M}(a)\right), t\right) dt.$$

Holding $\Pi(a)$ constant, the derivative of $W(a)$ with respect to $\bar{M}(a)$ is

$$\frac{dW}{d\bar{M}}|_{\Pi} = \frac{\Pi}{p^2 r}\left[c - \frac{\int_{\frac{c}{1-r}+\bar{M}}^{\infty}\frac{\partial Q(V,t)}{\partial V}dt}{\frac{\partial Q\left(V,\frac{c}{1-r}+\bar{M}\right)}{\partial V}dt} - \underbrace{\left(Q\left(V, \frac{c}{1-r}+\bar{M}\right) + \int_{\frac{c}{1-r}+\bar{M}}^{\infty}\frac{\partial Q(V,t)}{\partial V}\left(\frac{\frac{\partial Q\left(V,\frac{c}{1-r}+\bar{M}\right)}{\partial p}}{\frac{\partial Q\left(V,\frac{c}{1-r}+\bar{M}\right)}{\partial V}}\right)dt\right)}_{\geq 0}\right],$$

where one can use Assumption 1 to show

$$\begin{aligned}\int_{\frac{c}{1-r}+\bar{M}}^{\infty}\frac{\partial Q(V,t)}{\partial V}\left(\frac{\frac{\partial Q\left(V,\frac{c}{1-r}+\bar{M}\right)}{\partial p}}{\frac{\partial Q\left(V,\frac{c}{1-r}+\bar{M}\right)}{\partial V}}\right)dt &\geq \int_{\frac{c}{1-r}+\bar{M}}^{\infty}\frac{\partial Q(V,t)}{\partial V}\left(\frac{\frac{\partial Q(V,t)}{\partial p}}{\frac{\partial Q(V,t)}{\partial V}}\right)dt \\ &= \int_{\frac{c}{1-r}+\bar{M}}^{\infty}\frac{\partial Q(V,t)}{\partial p}dt = -Q\left(V(a), \frac{c}{1-r}+\bar{M}\right).\end{aligned}$$

- To establish threshold c^l, we note that $\frac{\partial Q(V,p)}{\partial V}$ being log-concave in p implies that

$$\phi_1(a,c) \equiv \frac{\int_{\frac{c}{1-r}+\bar{M}(a)}^{\infty}\frac{\partial Q(V(a),t)}{\partial V}dt}{\frac{\partial Q\left(V(a),\frac{c}{1-r}+\bar{M}(a)\right)}{\partial V}dt}$$

  is decreasing in c. For each c, denote

$$a^l(c) = \arg\min_a \phi_1(a,c),$$

  then $\phi_1\left(a^l(c), c\right)$ is decreasing in c by envelope theorem for arbitrary choice set Θ (Milgrom and Segal, 2002) given that the objective function $\phi_1(a,c)$ is differentiable in c. Then, we can define c^l as the unique solution to

$$c - \phi_1\left(a^l(c), c\right) = 0.$$

  The existence and uniqueness of c^l follows from the intermediate value theorem. By construction, for all $c < c^l$ and for all $a \in \Theta$, we have

$$\begin{aligned} c - \phi_1(a,c) &\leq c - \phi_1\left(a^l(c), c\right) \\ &< c^l - \phi_1\left(a^l\left(c^l\right), c^l\right) \\ &= 0, \end{aligned}$$

  Hence, $\frac{dW}{dM}|_{\Pi} < 0$ for all $c < c^l$ and $a \in \Theta$.

- To establish threshold c^h, let

$$\phi_2(a,c) \equiv \phi_1(a,c) + Q\left(V(a), \frac{c}{1-r} + \bar{M}(a)\right) + \underbrace{\int_{\frac{c}{1-r}+\bar{M}(a)}^{\infty} \frac{\partial Q(V(a),t)}{\partial V} \left(\frac{\frac{\partial Q\left(V(a), \frac{c}{1-r}+\bar{M}(a)\right)}{\partial p}}{\frac{\partial Q\left(V(a), \frac{c}{1-r}+\bar{M}(a)\right)}{\partial V}}\right)}_{\leq 0}.$$

For each c, denote

$$a^h(c) = \arg\max_a \left\{\phi_1(a,c) + Q\left(V(a), \frac{c}{1-r} + \bar{M}(a)\right)\right\},$$

then $\phi_1\left(a^h(c), c\right) + Q\left(V\left(a^h(c)\right), \frac{c}{1-r} + \bar{M}\left(a^h(c)\right)\right)$ is decreasing in c by envelope theorem (Milgrom and Segal, 2002). Then, we can define c^h as the unique solution to

$$c - \phi_1\left(a^h(c), c\right) - Q\left(V\left(a^h(c)\right), \frac{c}{1-r} + \bar{M}\left(a^h(c)\right)\right) = 0.$$

The existence and uniqueness of c^h follows from the intermediate value theorem as before. By construction, for all $c > c^h$ and for all $a \in \Theta$, we have

$$\begin{aligned}
c - \phi_2(a,c) &\geq c - \phi_1(a,c) - Q\left(V(a), \frac{c}{1-r} + \bar{M}(a)\right) \\
&\geq c - \phi_1\left(a^h(c), c\right) - Q\left(V\left(a^h(c)\right), \frac{c}{1-r} + \bar{M}\left(a^h(c)\right)\right) \\
&> c^h - \phi_1\left(a^h\left(c^h\right), c^h\right) - Q\left(V\left(a^h\left(c^h\right)\right), \frac{c^h}{1-r} + \bar{M}\left(a^h\left(c^h\right)\right)\right) \\
&= 0.
\end{aligned}$$

Hence, $\frac{dW}{dM}|_{\Pi} > 0$ for all $c > c^h$ and $a \in \Theta$.

Using the defined thresholds c^l and c^h, the remaining parts of the proof are the same as the proof of Proposition 2. ■